Quiz 4, Math 209, Winter 2007 **Name:** Solutions

Directions: Show ALL of your work to get credit. If you leave something out, then you may be penalized. No calculators. Good luck!

IMPORTANT: This quiz has two sides. Look at both!

1. [10 points] Find the volume of the solid lying under the paraboloid

$$z = 1 - \frac{x^2}{4} - \frac{y^2}{4}$$

and above the rectangle

$$R = [-1, 1] \times [0, 1] = \{(x, y) \mid -1 \le x \le 1,\, 0 \le y \le 1\}.$$

$$\int_{-1}^{1}\int_{0}^{1}\left(1 - \frac{x^2}{4} - \frac{y^2}{4}\right) dy\, dx =$$

$$= \int_{-1}^{1}\left(y - \frac{x^2}{4}y - \frac{y^3}{12}\right)\Big|_0^1 dx =$$

$$= \int_{-1}^{1}\left(1 - \frac{x^2}{4} - \frac{1}{12}\right) dx =$$

$$= \int_{-1}^{1}\left(\frac{11}{12} - \frac{x^2}{4}\right) dx$$

$$= \left(\frac{11}{12}x - \frac{x^3}{12}\right)\Big|_{-1}^{1} = \left(\frac{11}{12} - \frac{1}{12}\right) - \left(-\frac{11}{12} + \frac{1}{12}\right)$$

$$= \frac{11}{6} - \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$$

You could have also done

$$\int_{0}^{1}\int_{-1}^{1}\left(1 - \frac{x^2}{4} - \frac{y^2}{4}\right) dx\, dy$$

1

2. [10 points] Find the maximum and minimum values of

$$f(x, y) = x^2 + 2y^2$$

subject to the constraint $x^2 + y^2 = 1$.

$\nabla f(x,y) = \langle 2x, 4y \rangle$

$\lambda \nabla g(x,y) = \lambda \langle 2x, 2y \rangle = \langle 2x\lambda, 2y\lambda \rangle$

Need to solve

$$\left.\begin{array}{ll} (1) & 2x = 2x\lambda \\ (2) & 4y = 2y\lambda \\ (3) & x^2+y^2=1 \end{array}\right\} \quad \begin{array}{ll} (1) & x = x\lambda \\ (2) & 2y = y\lambda \\ (3) & x^2+y^2=1 \end{array}$$

Looking at (1) we see that either $x=0$ or $\lambda=1$.

If $x=0$, then using (3) we get $0+y^2=1 \Rightarrow y=\pm 1$.

(This also works in equation (2) by letting $\lambda=2$).

So we get the points $(0,1)$ and $(0,-1)$.

If $x \neq 0$ and $\lambda=1$, then equation (2) becomes $2y=y$. So, $y=0$. Then (3) gives $x^2+0^2=1$. So, $x=\pm 1$. Thus, we get the points $(1,0)$ and $(-1,0)$.

Check the points:

$$\left.\begin{array}{l} f(0,1)=2 \\ f(0,-1)=2 \end{array}\right\} \text{absolute maximums} \qquad \left.\begin{array}{l} f(1,0)=1 \\ f(-1,0)=1 \end{array}\right\} \text{absolute minimums.}$$