$\boxed{7.1}$

(1) Note that $0 = (-1)(0) = (-1)(-1+1) =$
$= (-1)^2 + (-1)(1) = (-1)^2 + (-1).$

So,
$$0 = (-1)^2 + (-1).$$

Thus,
$$0 + 1 = (-1)^2 + (-1) + 1.$$

So, $1 = (-1)^2.$

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(2) Let u be a unit of R.
Then there exists an element $x \in R$
with $ux = xu = 1$. Then,

$$(-u)(-x) = ux = 1$$

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and
$$(-x)(-u) = xu = 1.$$

So, $-u$ is a unit since $-x \in R$.

3 Let S be a subring of R and $u \in S$ be a unit of S. Then there exists $x \in S$ with $ux = xu = 1$. This equation also lives in R. So, u is a unit in R too.

<u>Converse</u>: If $S \subseteq R$ is a subring of R. ~~then a unit of R is a unit of S~~. Let $u \in S$. If u is a unit of R, then u is a unit of S.

NOT TRUE

<u>ex:</u> $R = \mathbb{R}$, $S = \mathbb{Z}$

$u = 2$ is a unit of $\mathbb{R}$ but not of $\mathbb{Z}$.

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11 Suppose $x^2 = 1$. Then, $x^2 - 1 = 0$. So, $(x-1)(x+1) = 0$. Since R is an integral domain, either $x - 1 = 0$ or $x + 1 = 0$. So, $x = 1$ or $x = -1$.

(14) (a) Suppose x is a nilpotent element. Then $x^m = 0$ for some $m \geq 1$. We may choose m to be minimal, that is, $x^k \neq 0$ for all k with $0 < k < m$.

If $x \neq 0$, then $x \cdot (x^{m-1}) = 0$. Since $x \neq 0$ and $x^{m-1} \neq 0$, we have that x is a zero divisor.

(b) Suppose x is nilpotent. Then $x^m = 0$ for some $m \geq 1$. Then,

$$(rx)^m \underset{\uparrow}{=} r^m x^m = r^m \cdot 0 = 0.$$

R is commutative

(c) Suppose x is nilpotent with $x^m = 0$. Then

$$\underbrace{(1-(-x))}_{(1+x)}\left(1 + (-x) + (-x)^2 + \cdots + (-x)^{m-1}\right)$$

$$= \begin{aligned} &1 + (-x) + (-x)^2 + \cdots + (-x)^{m-1} \\ &- (-x) - (-x)^2 - \cdots - (-x)^{m-1} - x^m \end{aligned}$$

$= 1.$ Thus, $(1+x)^{-1} = \left(1 + (-x) + (-x)^2 + \cdots + (-x)^{m-1}\right)$.