

Math 4900 - Final Exam Study Guide

Note: The test will have definition statements, computations, and proofs. If you learn the below topics then you will know what you need to know for the test.

For calculations, make sure to study the methods to determine whether M_n is prime or not below, that is, in particular, Examples 58 and 61 - 63. To practice Examples 58, and 61 - 63, try the examples in the notes and then construct your own examples. These are good test questions. Also make sure to learn the proofs that are listed in the third section below, as some of these exact proofs will be on the test.

Make sure to bring a calculator that isn't your phone to the test for calculations.

Final exam topics:

1. **Definitions / Statements** - You should be able to state the following.
 - (a) Def 25 - definition of a perfect number.
 - (b) Def 31 - definition of $\sigma(n)$
 - (c) Def 43 - Mersenne number / Mersenne prime
 - (d) Thm 44 - know the statement, you don't need to know the proof
 - (e) Thm 56 and Thm 57 - know the statements
 - (f) Def 59 - Lucas-Lehmer sequence s_n
 - (g) Know the statement of Thm 60
 - (h) Know the statement of Thm 64 (Euler)
 - (i) Know the statement of Thm 70
 - (j) Def 71 - $I(n)$
 - (k) Know the statement of Fact 73
 - (l) Def 76 - $\omega(n)$

2. **Calculations / Examples** - You should be able to do the following calculations.
 - (a) Ex 26 / Ex 27 - be able to compute if a number is perfect using Def 25.
 - (b) Ex 32 - be able to compute $\sigma(n)$.
 - (c) Show that Thm 44 is false by giving an example, such as M_{11} .
 - (d) Ex 58 - Be able to use Thm 56 and Thm 57 to test if M_n is prime or not.
 - (e) Ex 61,62,63 - Be able to use the Lucas-Lehmer test to test if M_n is prime or not.

- (f) Ex 72 - Be able to compute $I(n)$
- (g) Ex 77 - Be able to compute $\omega(n)$

3. **Proofs** - You should be able to prove the following.

- (a) Thm 29 / Thm 37 (Euclid's theorem) - you can know either version of the proof
- (b) Fact 33
- (c) Thm 35 - part 1 (only part 1, not the other parts)
- (d) Thm 41 (Euler)
- (e) Corollary 66 - in particular, if n is an odd perfect number then $n \equiv 1 \pmod{4}$.
- (f) Fact 75
- (g) Thm 78 - If n is an odd prime number then $\omega(n) \geq 2$. (You can use Lemma 79 without proof.)
- (h) Thm 80 - If n is an odd prime number then $\omega(n) \geq 3$. (You can use Lemma 79 without proof.)