(1)

$E = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$F = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5),$
$(5,1), (5,3), (5,5)\}$

$E \cap F = \{(1,5), (3,3), (5,1)\}$

$P(E \cap F) = \frac{3}{36} = \frac{1}{12} \approx 0.083$

$P(E) = \frac{5}{36}$

$P(F) = \frac{9}{36}$

$P(E)P(F) = \frac{45}{(36)^2} \approx 0.03472$

E and F are not independent
since $P(E \cap F) \neq P(E)P(F)$

(2) Let E be the event that the sum of the dice is 7 and F be the event that the red die is 2.

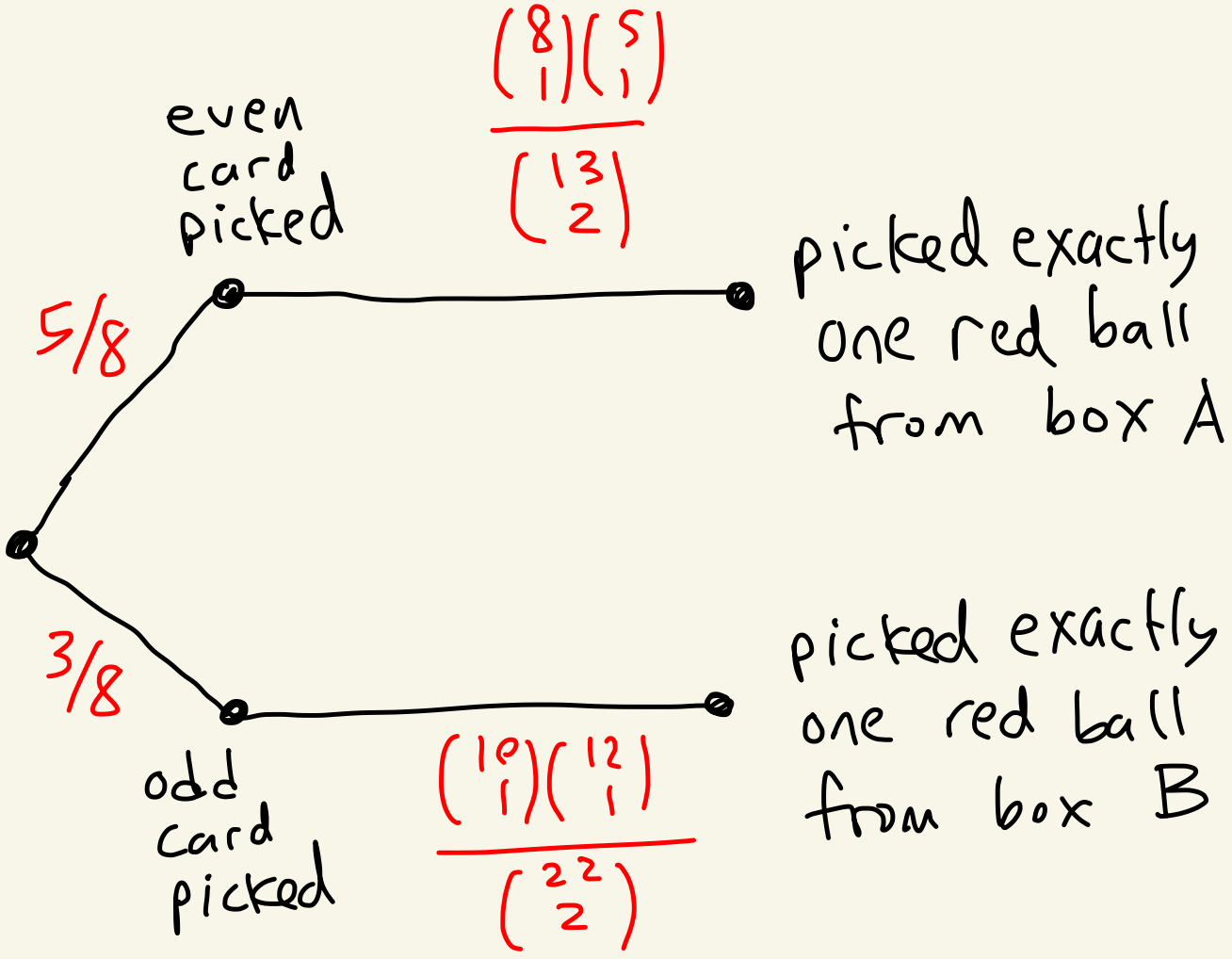
$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$F = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8)\}$

$E \cap F = \{(2,5)\}$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{1/36}{6/36} = \frac{1}{6}$$

(3)



Use total probability

$$\left(\frac{5}{8}\right)\cdot\left(\frac{8\cdot 5}{\frac{13\cdot 12}{2}}\right) + \left(\frac{3}{8}\right)\left(\frac{10\cdot 12}{\frac{22\cdot 21}{2}}\right)$$

$$= \frac{25}{78} + \frac{360}{1848} \approx 0.3205 + 0.1948$$

$$\approx 0.5153$$

(4)

See HW 3 problem 6 solution

(5)

(a) $P(X=-40) = P(\{(1,1)\}) = \frac{1}{16}$

(b) $P(X=-10)$

$= P(\{(2,1),(1,2),(3,1),(1,3),(4,1),(1,4)\}) = \frac{6}{16}$

(c) $P(X=20)$

$= P(\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4),$
$(4,2),(4,3),(4,4)\}) = 9/16$

(d)

$$E[X] = (-40)\left(\frac{1}{16}\right) + (-10)\left(\frac{6}{16}\right) + (20)\left(\frac{9}{16}\right)$$

$$= \frac{-40-60+180}{16}$$

$$= \frac{80}{16} = 5$$

So, $E[X] = \$5$

(6) Let A be the event that the sum of the dice is 7. Let B be the event that the sum of the dice is 4.

(a) $$\frac{P(A)}{P(A)+P(B)} = \frac{6/36}{6/36+3/36} = \frac{6}{9}$$

(b) $$\frac{P(B)}{P(B)+P(A)} = \frac{3/36}{3/36+6/36} = \frac{3}{9}$$

(c) $$E[x] = \left(\frac{6}{9}\right)(-5) + \left(\frac{3}{9}\right)(10)$$

$$= \frac{-30+30}{9} = 0$$

$$E[x] = \$0$$