1. Consider the experiment where you flip a coin 3 times. Let $X$ denote the number of tails that occur. Draw a picture of the probability function $p$ of $X$. Calculate $E[X]$ and $\text{Var}[X]$.

2. Consider the experiment where you roll two 4-sided dice. Let $X$ be the sum of the two dice.
   
   (a) Draw a picture of the probability function $p$ of $X$.
   
   (b) Draw a picture of the cumulative distribution function $F$ of $X$.
   
   (c) Calculate $E[X]$ and $\text{Var}[X]$ and $\sigma = \sigma_X$.

3. Consider the experiment where you roll two 4-sided dice. Let $X$ be the maximum of the two dice.
   
   (a) Draw a picture of the probability function $p$ of $X$.
   
   (b) Draw a picture of the cumulative distribution function $F$ of $X$.
   
   (c) Calculate $E[X]$ and $\text{Var}[X]$ and $\sigma = \sigma_X$.

4. You are interested in two games: game A and game B.

   - In game A, you pick a number between 1 and 100. A ball is drawn randomly from a box with balls that are numbered between 1 and 100. If the ball with your number is drawn then you win $74. Otherwise you lose $1.

   - In game B, there are four numbers to choose from. They are 1, 2, 3, and 4. You pick a number. Then a ball is drawn from a bag containing balls numbered 1, 2, 3, and 4. If your number is selected, then you win $2. Otherwise you lose $1.

Answer the following questions.

   (a) For each game let $X$ be the amount of money won or lost. Graph the probability function for $X$.

   (b) What is the expected value and variance of game A?
(c) What is the expected value and variance of game B?
(d) What game should you play?

5. Let $X$ be a discrete random variable. Let $\mu = E[X]$ and $\sigma^2 = \text{Var}[X]$.
   
   (a) Let $k$ be a positive real number. Use Chebyshev’s inequality to show that $P(|X \mu| \geq k\sigma) \leq \frac{1}{k^2}$.
   
   (b) Show that $P(|X \mu| \geq 2\sigma) \leq \frac{1}{4}$. [Note: This says that the probability that a data point is at least 2 standard deviations away from the mean (on either side) is at most 25%.

6. The binomial distribution applies when we are interested in the number of successes in a fixed number of Bernoulli trials. What if instead we studied how long it takes to get the first success in a series of Bernoulli trials. That is we look at the probability of having a string of failures (that is, multiple failures in a row) and then a success. More specifically, let $0 < p < 1$ and $q = 1 - p$. Consider the experiment where we do consecutive independent Bernoulli trials with probability $p$ of success and $q$ of failure. We repeat the experiment until we get the first success and then we stop.

   (a) What is a sample space $S$ for this experiment? Let $X$ be the number of trials until the first success occurs. Find a formula for $P(X = k)$. Note: $X$ is called a Geometric random variable.
   
   (b) Sketch the probability function $p(k) = P(X = k)$ when the probability of success is $\frac{1}{2}$.

   (c) Show that $E[X] = \frac{1}{p}$ and $\text{Var}[X] = \frac{1-p}{p^2} = \frac{q}{p^2}$. 