ANSWERS TO HW #3 (EXTRA PROBLEMS)

Recall the following theorem from class: Let S be a sample space of a repeatable experiment. Let A and B be mutually exclusive events in S, that is their intersection is empty. Suppose further that each time you repeat S the experiment is independent of the previous times you did the experiment S. Suppose that we repeat S over and over until either A occurs or B occurs. Then the probability that A occurs before B occurs is

$$\frac{P(A)}{P(A) + P(B)}$$

1. Let S be a sample space for rolling two 6-sided dice. Let A be the event that the sum of the dice is 6 and B be the event that the sum of the dice is 7. Then A and B are mutually exclusive. And each time you roll the two dice the outcome is independent of the previous times you rolled the dice. Thus we can use the theorem.

   (a) The probability that you roll a sum of 6 before you roll a sum of 7 is
   $$\frac{P(A)}{P(A) + P(B)} = \frac{5/36}{5/36 + 6/36} = \frac{5}{11}$$

   (b) The probability that you roll a sum of 7 before you roll a sum of 6 is
   $$\frac{P(B)}{P(B) + P(A)} = \frac{6/36}{6/36 + 5/36} = \frac{6}{11}$$

2. Let S be a sample space reflecting the experiment of randomly selecting a card from a standard 52 card deck. Let A be the event that the card is an ace card. Let B be the event that the card is a face card. Then A and B are mutually exclusive. And each time you pick a card from the deck after you reshuffle it, the outcome is independent of the previous times you selected a card. (This is because you replace the chosen card and reshuffle each time.) Thus we can use the theorem. Note that there are 4 aces and $4 + 4 + 4 = 12$ face cards.

(a) The probability that an ace comes up before a face card is

$$\frac{P(A)}{P(A)+P(B)} = \frac{4/52}{4/52+12/52} = \frac{4}{16} = \frac{1}{4}$$

(b) The probability that a face card comes up before an ace is

$$\frac{P(B)}{P(B)+P(A)} = \frac{12/52}{12/52+4/52} = \frac{12}{16} = \frac{3}{4}$$