

## Math 474 - Homework # 1

### Sets and Probability Spaces

- (a) Consider the experiment of rolling two 6-sided dice. What is a sample space  $S$  and a probability function  $P$  for this experiment? (b) Let  $A$  be the event that the sum of the dice is 2. Let  $B$  be the event that the sum of the dice is 4. What are the elements of  $A$  and  $B$ ? (c) Calculate  $A \cup B$ ,  $A \cap B$ ,  $\overline{A}$ ,  $\overline{B}$ . (d) How do these sets relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events from parts b and c.
- Consider the experiment of rolling two 6-sided dice. (a) What is a sample space  $S$  and a probability function  $P$  for this experiment? (b) Let  $A$  be the event that at least one of the dice is even. Let  $B$  be the event that both of the dice are even. Let  $C$  be the event that one of the dice is odd. Let  $D$  be the event that both of the dice are odd. List the elements of  $A$ ,  $B$ ,  $C$ , and  $D$ . (c) Calculate  $A \cap C$ ,  $\overline{A}$ ,  $B \cap D$ ,  $B \cup D$ ,  $\overline{B}$ ,  $\overline{D}$ . (d) How do these sets relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.
- Consider the experiment of flipping a coin 4 times in a row. (a) What is a sample space  $S$  and a probability function  $P$  for this experiment? (b) Let  $A$  be the event that a head occurs on the first flip. Write out the elements of  $A$ . Let  $B$  be the event that a tails occurs on the second flip and the fourth flip. Write out the elements of  $B$ . (c) Calculate  $A \cup B$ ,  $A \cap B$ ,  $\overline{A}$ ,  $\overline{B}$ . (d) How do these sets relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.
- Consider the experiment of rolling a weighted 4-sided dice. Weighted means that the probabilities aren't equal for each number on the dice. Suppose that through experimentation you discover that the 1 occurs two times for every eight rolls of the dice, a 2 occurs two times for every eight rolls of the dice, a 3 occurs three times for every eight rolls of the dice, and a 4 occurs one time for every eight rolls of the dice. (a) What is a sample space  $S$  and a probability function  $P$  for this experiment?

- (b) Let  $A$  be the event of rolling a 1 or a 3. What are the elements of  $A$  and what is the probability of  $A$ ? (c) Let  $B$  be the event of rolling a 1 or a 2 or a 3. What are the elements of  $B$  and what is the probability of  $B$ ?
5. Suppose that two 4-sided dice are thrown.
- (a) What is the probability that one of the dice shows a 2?
  - (b) What is the probability that the sum of the dice is 4?
  - (c) What is the probability that the sum of the dice is either a 5 or a 7?
6. Suppose that two 8-sided dice are thrown, one green and one red. What is the probability that the red die has a larger value than the green die?
7. Consider an bag that contains three balls: 1 white balls, 1 red ball, and 1 green ball. Suppose you want to construct a probabilistic model of the experiment where you randomly (without looking) choose 1 ball from the bag and record the result. How could you do such a thing? What is the sample space  $S$ ? Describe the probability function  $P$ . What is the probability of choosing a white ball? Of choosing a red ball?
8. Suppose we take the same setup as problem 7. But we change it to the experiment where you randomly choose 2 balls from the bag. What is a possible sample space  $S$  and probability function  $P$  for this model? What is the probability of choosing one white ball and one red ball?
9. Consider the experiment where one 4-sided dice is thrown continually until a 2 is rolled. Construct a probability space that models this experiment. Verify that you have a probability space.
10. Consider the experiment where a coin is flipped continually until a head occurs. Construct a probability space that models this experiment. Verify that you have a probability space.