**Roulette**

Sample Space

\[ S = \{0, 00, 1, 2, 3, \ldots, 35, 36\} \]

\[ 18| = 38 \]

each outcome is equally likely

**Single number bet**

Bet on \#12, \( E = \{12\} \)

\[ P(E) = \frac{1}{38} \]

The casino pays 35:1 on this type of bet.

Suppose you place $1 on 12. Let \( X \) be the amount won or lost.

\[ X(w) = \begin{cases} 
35 & \text{if } w = 12 \\
-1 & \text{if } w \neq 12 
\end{cases} \]

\[ E[X] = (35) \cdot P(X = 35) + (-1) \cdot P(X = -1) \]

\[ = (35) \cdot \frac{1}{38} + (-1) \cdot \left( \frac{37}{38} \right) \approx -0.0526 \]

On average in the limit, you lose that much per game.

What if instead the casino played you the true odds odds against \( E = \frac{P(E)}{P(\overline{E})} = \frac{37}{38} \)?

True odds are 37:1

\[ X(w) = \begin{cases} 
37 & \text{if } w = 12 \\
-1 & \text{if } w \neq 12 
\end{cases} \]

\[ E[X] = (37) \cdot \frac{1}{38} + (-1) \cdot \left( \frac{37}{38} \right) = 0 \]

# Think of expected value as average.
Dozen Bet

$E = \{3, 6, 12\} = \{26, 28, 29, 30, 31, 32, 33, 34, 35, 36\}$

Casino pays 2:1

Let $X = \text{amount won or lost for } $1\text{ bet.}$

$X(w) = \begin{cases} 
2 & \text{if } 26 \leq w \leq 36 \\
-1 & \text{otherwise} 
\end{cases}$

$E[X] = \left(\frac{12}{38}\right) \cdot 2 + \left(\frac{26}{38}\right) \cdot (-1) \approx -0.0526$

If you make 38 bets on average,

- win 12 of them $24
- lose 26 of them $26

$\frac{12}{38}$
Game of Craps

Suppose we make a pass line bet and 4 is the come out roll, so 4 is the Point.

What's the probability we win or lose on the following rolls?

Sample Space: \( S = W \cup L \)

\[ W = \{ (4), (\text{not y, 4}), (\text{not y, not y, 4}), \ldots \} \]

\[ L = \{ (7), (\text{not y, 7}), (\text{not y, not y, 7}), \ldots \} \]

Point is 4

\[ P(W) = \frac{2}{36} + \frac{2}{36} \cdot \frac{2}{36} + \frac{2}{36} \cdot \frac{2}{36} \cdot \frac{2}{36} + \cdots = \frac{3}{9} \]

\[ P(L) = \frac{6}{36} \left[ 1 + \frac{2}{36} + \left( \frac{2}{36} \right)^2 + \left( \frac{2}{36} \right)^3 + \cdots \right] = \frac{6}{9} \]

\[ P(W) = \frac{3}{9} \quad \text{and} \quad P(L) = \frac{6}{9} \]

If \(|x| < 1\), then

\[ 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} \]

Geometric Sum
Probabilities after come-out roll and a point has been made

<table>
<thead>
<tr>
<th>Point</th>
<th>Probability of rolling point before 7</th>
<th>Probability of rolling a 7 before point</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3/9</td>
<td>6/9</td>
</tr>
<tr>
<td>5</td>
<td>4/10</td>
<td>6/10</td>
</tr>
<tr>
<td>6</td>
<td>5/11</td>
<td>6/11</td>
</tr>
<tr>
<td>8</td>
<td>5/11</td>
<td>6/11</td>
</tr>
<tr>
<td>9</td>
<td>4/10</td>
<td>6/11</td>
</tr>
<tr>
<td>10</td>
<td>3/9</td>
<td>6/9</td>
</tr>
</tbody>
</table>

how the game works:

- 7 or 11 is a **natural win**
- 2, 3, 12 is **craps lose**
- point is 4
  - 7 rolled before a 4 lose
  - 4 rolled before a 7 win
- point is 5
  - 7 rolled before a 5 lose
  - 5 rolled before a 7 win
- exclude 7, 11, 12
  - 7 rolled before 10 lose
  - 10 rolled before 7 win
HW #3

4) A box contains 7 red and 13 blue balls. Two balls are selected at random, one at a time, and are discarded w/o their colors being seen. If a third ball is drawn and observed to be red, what is the probability that the first two balls were blue?

Let RR, RB, BB be the events where the first two balls were red/red, red/blue or blue/red, or blue/blue.

Let R be the event the third ball is red.

Want \( P(BR|R) \)

\[ P(BB|R) = \frac{P(BB \cap R)}{P(R)} \]

\[ \text{Step 1}\]

\[ = \frac{P(RBB)}{P(R)} \]

\[ \text{Step 2}\]

\[ \approx 0.46 \]

\[ \frac{13 \cdot 12}{20 \cdot 19} \cdot \frac{\frac{7}{18}}{13 \cdot 12} \]

\[ \frac{13 \cdot 12}{20 \cdot 19} \cdot \frac{7}{18} = \frac{13 \cdot 12}{20 \cdot 19} \cdot \frac{6}{18} = \frac{7}{18} \cdot \frac{5}{18} \]

\[ \approx 0.46 \]

Prob. you pull out a red ball given you already pulled 2 blue balls.

\[ \frac{13}{20} \cdot \frac{12}{19} \]

\[ \begin{array}{c}
\text{RR} \\
\text{RB} \\
\text{BB}
\end{array} \]

\[ \frac{13}{20} \cdot \frac{12}{19} = \frac{13 \cdot 12}{20 \cdot 19} \]

\[ \frac{13}{20} \cdot \frac{12}{19} = \frac{13 \cdot 12}{20 \cdot 19} \]
St. Petersburg Paradox  Just for fun

A casino offers a game where a coin is flipped. You first put $2. You have to double the pot (win amount) each time a head appears. The first time a tail appears