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Proposition Let (S, Ω, P) be a probability space. Let E and F be events. Then

① $P(\overline{E}) = 1 - P(E)$

② If $E \subseteq F$, then $P(E) \leq P(F)$.

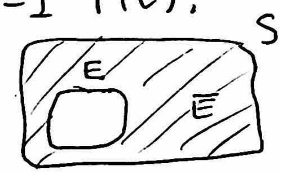
③ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

④ If $E \cap F = \phi$, then $P(E \cup F) = P(E) + P(F)$

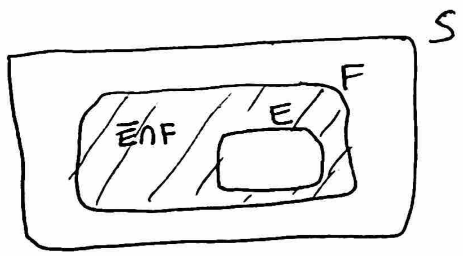
proof:

① Note that $S = E \cup \overline{E}$. and $E \cap \overline{E} = \phi$.

Thus, $\underset{\boxed{\text{axiom 4}}}{1 = P(S)} = P(E \cup \overline{E}) \underset{\boxed{\text{axiom 5}}}{=} P(E) + P(\overline{E})$, Thus, $P(\overline{E}) = 1 - P(E)$.



② Since $E \subseteq F$ we can write $F = E \cup (\overline{E} \cap F)$



And E and $\overline{E} \cap F$ are disjoint.

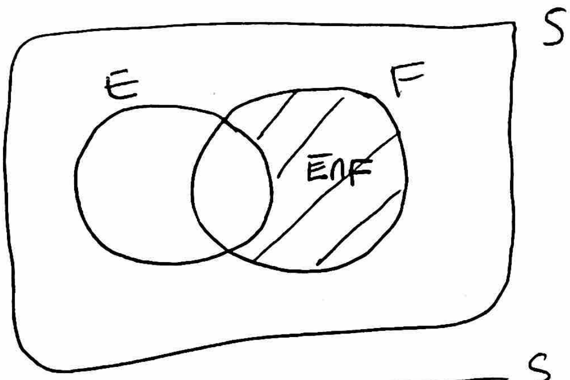
Thus, $P(F) = P(E \cup (\overline{E} \cap F))$

$$\underset{\boxed{\text{axiom 5}}}{=} P(E) + \underbrace{P(\overline{E} \cap F)}_{\geq 0}$$

So, $P(F) \geq P(E)$ [because $P(\overline{E} \cap F) \geq 0$].

③ Note that $E \cup F = E \cup (\overline{E} \cap F)$. And E and $\overline{E} \cap F$ are disjoint, Thus, by axiom 5,

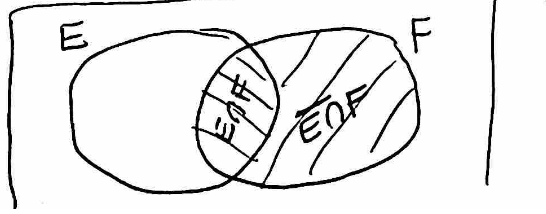
$P(E \cup F) = P(E) + P(\overline{E} \cap F)$,



Furthermore,

$F = (E \cap F) \cup (\overline{E} \cap F)$.

and $E \cap F$ and $\overline{E} \cap F$ are disjoint.



Hence, by axiom 5,

$P(F) = P(E \cap F) + P(\overline{E} \cap F)$.

So, $P(\overline{E} \cap F) = P(F) - P(E \cap F)$.

Thus, $P(E \cup F) = P(E) + P(\overline{E} \cap F)$

$= P(E) + P(F) - P(E \cap F)$.