

Math 4680 - Homework # 7

Path Connected

- For the following sets S , (i) determine whether or not the set is path-connected, and (ii) determine whether or not the set is a region (or domain). Recall that a region is an open, path-connected subset of \mathbb{C} .
 - $S = \{z \mid 1 \leq |z| \leq 2\}$
 - $S = \{z \mid |z| < 2 \text{ and } |\operatorname{Re}(z)| > 1\}$
 - $S = \{z \mid \operatorname{Re}(z) < 1\}$
 - $S = \{z \mid |z| < 1 \text{ or } |z| > 2\}$
- Let A be an open, path-connected subset of \mathbb{C} . Let $f : A \rightarrow \mathbb{C}$ be analytic on A . Prove that if $f(z)$ is real for all $z \in A$, then f is constant on A .