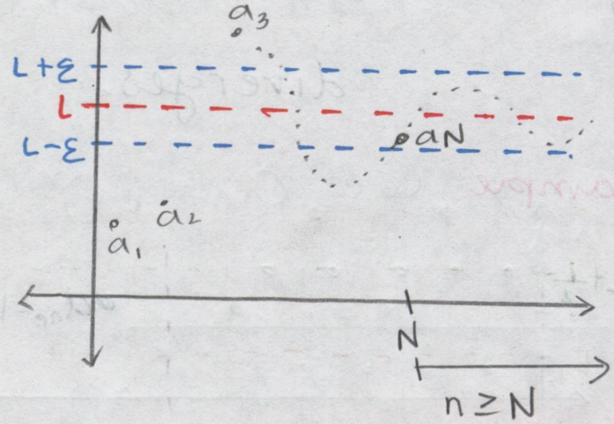
Last Time: $\lim_{n\to\infty} a_n = L$

if for every $\varepsilon > 0$ $\exists N > 0$ such that if $n \geq N$ then $|a_n - L| < \varepsilon$



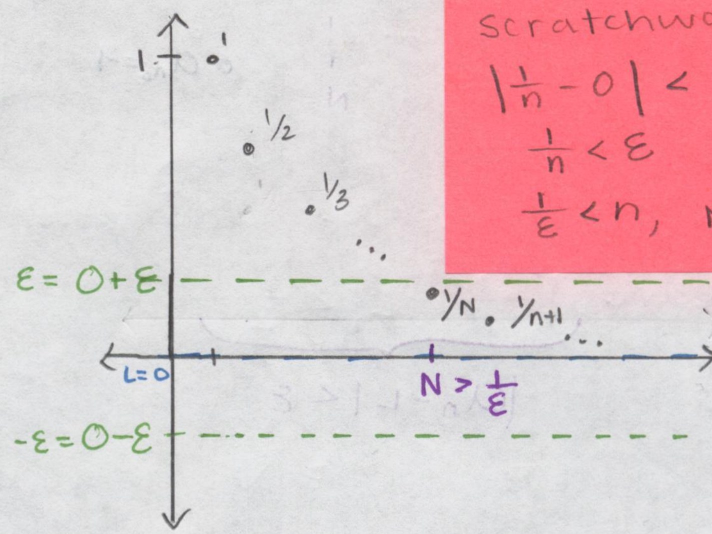
Example: $\lim_{n\to\infty} \frac{1}{n} = 0$

proof: Let $\varepsilon > 0$

pick N where $N > \frac{1}{\varepsilon}$

Then if $n \geq N$ then

$$\left|\frac{1}{n} - 0\right| \underset{\substack{\uparrow \\ n>0}}{=} \frac{1}{n} \underset{\substack{\uparrow \\ n\geq N}}{\leq} \frac{1}{N} \underset{\substack{\uparrow \\ N>\frac{1}{\varepsilon}}}{<} \varepsilon \quad \blacksquare$$



scratchwork

$\left|\frac{1}{n} - 0\right| < \varepsilon$

$\frac{1}{n} < \varepsilon$

$\frac{1}{\varepsilon} < n, \quad N > \frac{1}{\varepsilon}$

Example: Let's show that $\lim_{n\to\infty} \frac{n}{n+1} = 1$

proof: Let $\varepsilon > 0$

For any $n > 0$, note that

$$(*) \quad \left|\frac{n}{n+1} - 1\right| = \left|\frac{n}{n+1} - \frac{n+1}{n+1}\right| = \left|\frac{-1}{n+1}\right| \underset{\substack{\uparrow \\ n>0}}{=} \frac{1}{n+1}$$

scratchwork

$\frac{1}{n+1} < \varepsilon$

$\frac{1}{\varepsilon} < n+1$

$\frac{1}{\varepsilon} - 1 < n$

Note that $\frac{1}{n+1} < \varepsilon$ iff $\frac{1}{\varepsilon} < n+1$ iff $\frac{1}{\varepsilon} - 1 < n$.

Pick N so that $N > \frac{1}{\varepsilon} - 1$. If $n \geq N > \frac{1}{\varepsilon} - 1$, then by the previous equations (*) we have:

$$\left|\frac{n}{n+1} - 1\right| = \frac{1}{n+1} < \varepsilon \quad \blacksquare$$