note: ways to prove that some bound on a set is the inf/sup of the set

(1) use useful inf/sup fact

(2) Use def. Let $S \subseteq \mathbb{R}$, $S \neq \emptyset$

- $b = \sup(S)$ iff

  (i) $x \leq b$ for $x \in S$

  (ii) $b \leq c$ $\forall$ upper bounds c of S

- $b = \inf(S)$ iff

  (i) $b \leq x$ for $x \in S$

  (ii) $c \leq b$ $\forall$ lower bounds c of S

Completeness Axiom

Let $S \subseteq \mathbb{R}$ with $S \neq \emptyset$

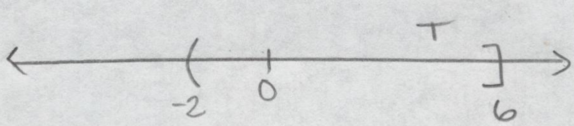
If S is bounded from above, then the supremum of S exists.

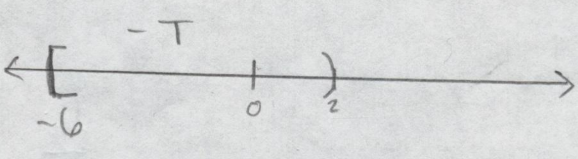
Theorem: If $T \subseteq \mathbb{R}$ and $T \neq \emptyset$

If T is bounded from below then the infimum of T exists

Proof: Let $T \subseteq \mathbb{R}$, $T \neq \emptyset$, and T be bounded from below

Ex: $T = (-2, 6]$





Let $-T = \{-x \mid x \in T\}$