Heine-Borel

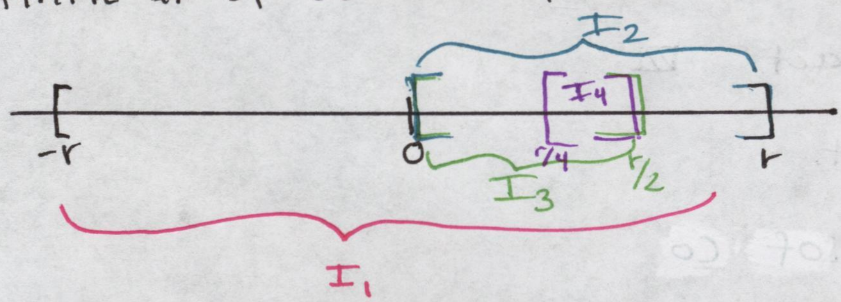
Let $K \subseteq \mathbb{R}$, K is compact iff K is closed and bounded

proof:

($\Leftarrow$) Suppose K is closed and bounded

summary of last time:

- Let $G = \{G_\alpha\}$ be an open cover of K
- Suppose K is not covered in a finite # of sets from G. } proof by contradiction
- created a sequence of closed intervals $\dots \subseteq I_3 \subseteq I_2 \subseteq I_1 = [-r, r]$ where $K \subseteq [r, r]$ and $K \cap I_i \neq \emptyset$ and $K \cap I_i$ is not covered by a finite # of elements from G.



- let $I_n = [a_n, b_n]$

  Let $\xi = \sup\{a_k \mid k \geq 1\}$

  we showed $\xi = \inf\{b_k \mid k \geq 1$

  we showed ξ is a limit point of K

  we showed $\xi \in \bigcap_{n=1}^{\infty} I_n$

continuation from last time:

- since K is closed, K contains all its limit points.

So, $\xi \in K$. So there exists $G_\xi \in G$ where $\xi \in G_\xi$