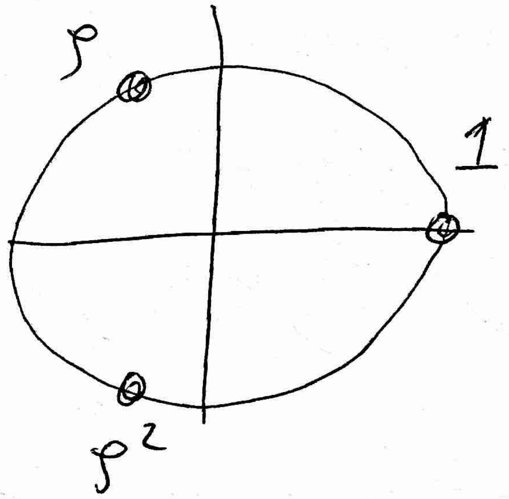
Hw 2

Better way to do #2, 9, 10 + extra part for #2.

(2) $U_3 = \{1, \rho, \rho^2\}$ where $\rho = e^{2\pi i/3}$ and $\rho^3 = 1$.



1 has order 1 since $1 = 1$.

ρ has order 3 since
$\rho \neq 1$
$\rho^2 \neq 1$
$\rho^3 = 1$

ρ^2 has order 3 since
$\rho^2 \neq 1$
$(\rho^2)^2 = \rho^4 = \rho \neq 1$
$(\rho^2)^3 = \rho^6 = \rho^3\rho^3 = 1 \cdot 1 = 1$.

New problem:

Calculate the order of ρ^5 in U_{10}. Do the same for ρ^3.
Here $\rho = e^{2\pi i/10}$ and $\rho^{10} = 1$, $U_{10} = \{1, \rho, \rho^2, \rho^3, \rho^4, \rho^5, \rho^6, \rho^7, \rho^8, \rho^9\}$

ρ^5)

$\rho^5 \neq 1$
$(\rho^5)^2 = \rho^{10} = 1$.
So, ρ^5 has order 2.

ρ^3)

$\rho^3 \neq 1$
$(\rho^3)^2 = \rho^6 \neq 1$
$(\rho^3)^3 = \rho^9 \neq 1$
$(\rho^3)^4 = \rho^{12} = \rho^2 \neq 1$
$\rho^{10} = 1$
$(\rho^3)^5 = \rho^{15} = \rho^{10}\rho^5 = \rho^5$
$(\rho^3)^6 = \rho^{18} = \rho^{10}\rho^8 = \rho^8$
$(\rho^3)^7 = \rho^{21} = \rho^{10}\rho^{10}\rho = \rho$
$(\rho^3)^8 = \rho^{24} = \rho^{10}\rho^{10}\rho^4 = \rho^4$
$(\rho^3)^9 = \rho^{27} = \rho^{10}\rho^{10}\rho^7 = \rho^7$
$(\rho^3)^{10} = (\rho^{10})^3 = 1^3 = 1$

$\boxed{\rho^3 \text{ has order } 10}$

New problem:

Calculate the subgroup generated by ρ^5 in U_{10}. Same problem for ρ^3.

We Use the calculations from the previous problem.

$$\langle \rho^5 \rangle = \{1, \rho^5\}$$

$$\langle \rho^3 \rangle = \{1, \rho^3, (\rho^3)^2, (\rho^3)^3, (\rho^3)^4, \ldots, (\rho^3)^9\}$$

$$= \{1, \rho^3, \rho^6, \rho^9, \rho^2, \rho^5, \rho^8, \rho, \rho^4, \rho^7\}$$

$$= U_{10}.$$

So, ρ^3 generates U_{10}

ρ^5 does not generate U_{10}.
But it does generate a subgroup of size (order) 2.

$$\langle \rho^5 \rangle = \{1, \rho^5\}$$

(9) $\rho = e^{2\pi i/6} = e^{\pi i/3}$ and $\rho^6 = 1$

$U_6 = \{1, \rho, \rho^2, \rho^3, \rho^4, \rho^5\}$

$\langle e^{2\pi i/3} \rangle = \langle \rho^2 \rangle = \{1, \rho^2, (\rho^2)^2\} = \{1, \rho^2, \rho^4\}$

since $(\rho^2)^3 = \rho^6 = 1$

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(10) $\rho = e^{2\pi i/8} = e^{\pi i/4}$, and $\rho^8 = 1$

$U_8 = \{1, \rho, \rho^2, \rho^3, \rho^4, \rho^5, \rho^6, \rho^7\}$

$\langle e^{3\pi i/4} \rangle = \langle \rho^3 \rangle = \{1, \rho^3, (\rho^3)^2 = \rho^6, (\rho^3)^3 = \rho^9 = \rho,$
$(\rho^3)^4 = \rho^{12} = \rho^4, (\rho^3)^5 = \rho^{15} = \rho^7, (\rho^3)^6 = \rho^{18} = \rho^2,$
$(\rho^3)^7 = \rho^{21} = \rho^5\} = \{1, \rho^3, \rho^6, \rho, \rho^4, \rho^7, \rho^2, \rho^5\} = U_8$

ex: $\rho^{21} = \rho^8 \rho^8 \rho^5 = 1 \cdot 1 \cdot \rho^5 = \rho^5$

stopped at $(\rho^3)^7$ since $(\rho^3)^8 = (\rho^8)^3 = 1 \cdot 1 \cdot 1 = 1$