Homework #10

① For each matrix A do the following:

(i) Find the eigenvalues of A.

(ii) Find a basis for each eigenspace $E_\lambda(A)$.

(iii) For each eigenvalue, compute it's algebraic and geometric multiplicity

(iv) determine whether or not A is diagonalizable, and if so find P where $P^{-1}AP$ is diagonal.

(a) $A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{pmatrix}$

(c) $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ (d) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

(e) $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$

② Let A be an $n \times n$ matrix and λ be an eigenvalue of A. Prove that $E_\lambda(A)$ is a subspace of $\mathbb{R}^n$.

③ Let A be an $n \times n$ matrix. Suppose that λ is an eigenvalue of A with corresponding eigenvector $\vec{x}$. Find a formula for $A^n \vec{x}$ for any $n = 1, 2, 3, 4, \ldots$