

Vector Spaces

Def: A field F is a set with two operations $+$ and \cdot such that for each pair of elements x and y in F there are unique elements $x+y$ and $x \cdot y$ in F for which the following conditions hold for all elements a, b, c in F .

① $a+b = b+a$

② $ab = ba$

③ $(a+b)+c = a+(b+c)$

④ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

⑤ There exist distinct elements 0 and 1 in F such that $0+a = a+0 = a$ and $1 \cdot a = a \cdot 1 = a$.

⑥ For each element a there exist ~~an~~ elements d and $e \in F$ such that $a+d = d+a = 0$ and $a \cdot e = e \cdot a = 1$.

⑦ $a \cdot (b+c) = a \cdot b + a \cdot c$
 $(b+c) \cdot a = b \cdot a + c \cdot a$

d is usually written as $-a$. e is usually written as a^{-1} .

Ex: $F = \mathbb{R}$ the set of real numbers

Ex: $F = \mathbb{C}$ the set of complex numbers