Math 2120 - Spring 2020 - Final

Directions:

1. Pick a consecutive 2.5-hour window to take this exam, such as 12pm - 2:30pm. You may only use 2.5 hours of consecutive time. Do not split the time (like 12-1pm and then 5-6:30pm).

2. You can only use your mind and one page of formulas to take this exam. No help from any sources or people. No books, no notes, no online, etc.

3. No calculators.

4. Use blank paper (like printer paper) if you have it please.

5. On the first page of your exam, before any of your solutions, put these three things:

   (a) Your name.

   (b) The time period that you chose. (Such as 12pm - 2:30pm on Tuesday)

   (c) Copy this statement and then sign your signature after it:

   "Everything on this test is my own work. I did not use any sources or talk to anyone about this exam." _____**your signature**_____

6. After your name and the above statement with signature, start putting your solutions to the problems. Please put them in order. That is, first problem 1, then problem 2, etc. You can put each one on its own page.

7. Scan and email to me by Wednesday the 13th at 12pm.

The problems are on the next page.

1. Do parts (a) and (b) below.

   (a) Calculate $\int x^{-2}\ln(x)\,dx$

   (b) Determine whether the following integral converges or diverges: $\int_1^\infty x^{-2}\ln(x)\,dx$

2. Calculate $\int \sqrt{4-x^2}\,dx$

   [*Hint - You might need this formula near the end:* $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$]

3. Calculate $\int \frac{dx}{(x-2)(x+1)}$

4. What does the following series add up to? That is, find the number S where

$$S = \sum_{k=2}^{\infty} 3 \cdot \frac{5^k}{7^{k+1}}$$

5. Does the following series converge or diverge? Make sure to explain what test you are using and why the conditions of the test are satisfied.

$$\sum_{k=1}^{\infty} \frac{k^2+1}{2k^2-1}$$

6. (a) Use a test to show that the following sum converges

$$\sum_{k=1}^{\infty} \frac{2^k}{k!}$$

   (b) What does the above sum converge to? [Hint: It isn't a geometric series.]

7. Find the first three terms in the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 1$.

8. Find parametric equations for the line through the points $(-4, -6, 1)$ and $(-2, 0, -3)$.

9. (a) Sketch the graph of the curve given by $\mathbf{r}(t) = \langle 1+t, t^2\rangle$ by plotting the points corresponding to $t = -3, -2, -1, 0, 1, 2, 3$. Label your points and indicate the direction of motion with arrows on the curve.

   (b) Calculate $\mathbf{r}'(t)$ at $t = 1$. Draw a picture of $\mathbf{r}'(1)$ on your graph for part (a).

   (c) Find a vector that is orthogonal/perendicular to $\mathbf{r}'(1)$ and show why it is orthogonal/perpendicular to it.

There is another page after this one.

10. (a) Sketch the graph of $r = \sin(2\theta)$.

Make sure to make a table of plotted values so I can see your work. And label the graph with arrows to show the direction of motion. Note that $\frac{\sqrt{2}}{2} \approx 0.71$ *and* $\frac{\sqrt{3}}{2} \approx 0.87$.

(b) Find the area inside one of the leafs of the graph.