Test 1, Math 2130, Fall 2019 **Name:** Solutions

Directions: Show all of your work to get credit. <u>No calculators</u>. Good luck!

1. [5 points] Find an equation of the plane passing through the points $(0, 0, 3)$, $(1, 0, -6)$, and $(1, 2, 3)$.

$P = (0,0,3)$

$Q = (1,0,-6)$

$R = (1,2,3)$

$\overrightarrow{PR} = \langle 1-0, 2-0, 3-3 \rangle = \langle 1,2,0 \rangle$

$\overrightarrow{PQ} = \langle 1-0, 0-0, -6-3 \rangle = \langle 1,0,-9 \rangle$

$$\vec{n} = \overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 0 \\ 1 & 0 & -9 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & 0 \\ 0 & -9 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 0 \\ 1 & -9 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-18) - \hat{j}(-9) + k(-2)$$

$$= -18\hat{i} + 9\hat{j} - 2k$$

<u>plane equation</u>

$\vec{n} = \langle -18, 9, -2 \rangle$

point on plane $(0,0,3)$

$$-18(x-0) + 9(y-0) - 2(z-3) = 0$$

$$-18x + 9y - 2z + 6 = 0$$

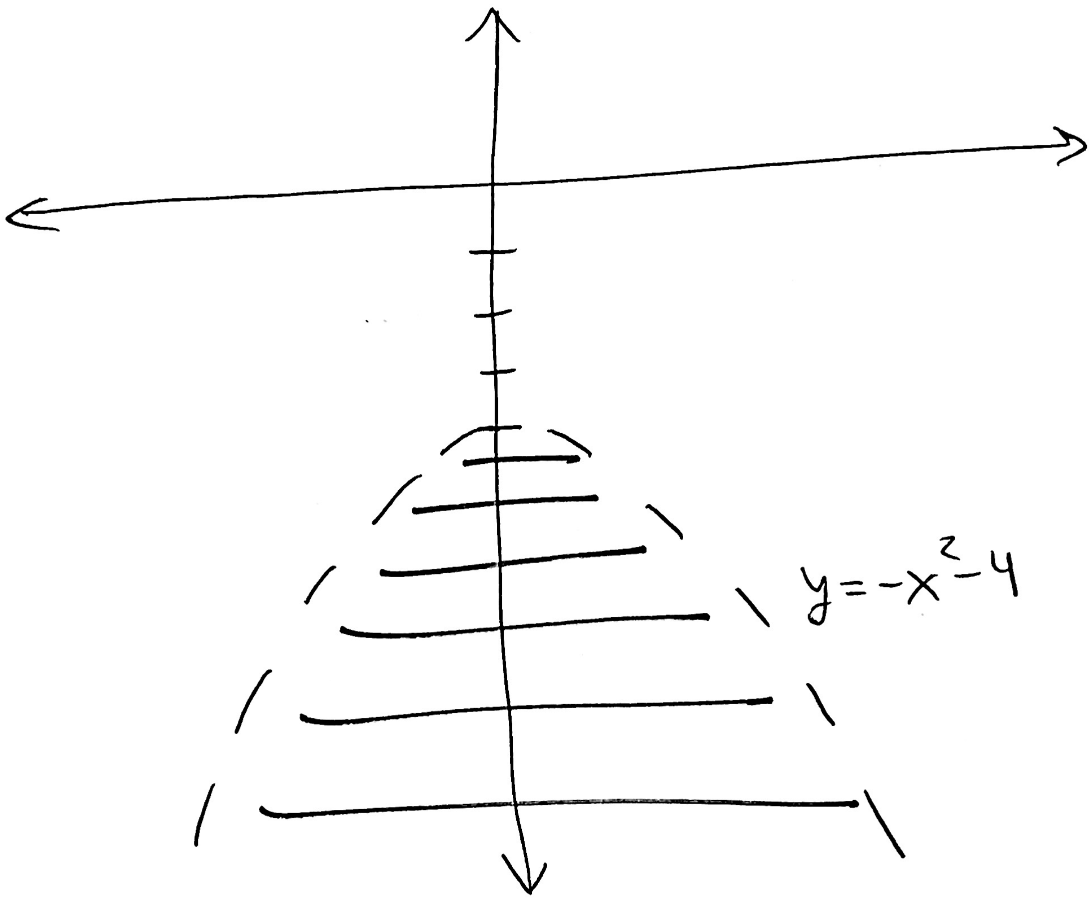
2. [5 points] Find and sketch the domain of

$$f(x,y) = \frac{2x}{\sqrt{-x^2 - y - 4}}$$

Need $-x^2 - y - 4 > 0$.

So, $\boxed{-x^2 - 4 > y}$.

The domain of f consists of all (x, y) with $-x^2 - 4 > y$.



3. [10 points - 5 each] If the limit exists, calculate it. If the limit does not exist show why.

(a) $\lim_{(x,y)\to(-1,5)} \frac{e^{xy}}{x-y}$ $= \frac{e^{(-1)(5)}}{-1-5} = \boxed{-\frac{1}{6}e^{-5}}$

(b) $\lim_{(x,y)\to(0,0)} \frac{4xy}{3x^2+y^2}$

If $x=0$, then $\frac{4xy}{3x^2+y^2} = \frac{0}{y^2} = 0 \to 0$ as $y \to 0$.

If $y=0$, then $\frac{4xy}{3x^2+y^2} = \frac{0}{3x^2} = 0 \to 0$ as $x \to 0$

If $y=x$, then $\frac{4xy}{3x^2+y^2} = \frac{4x^2}{3x^2+x^2} = 1 \to 1$ as $x \to 0$, $y \to 0$.

Since we get diffferent values by approaching $(0,0)$ an the x-axis and the curve $y=x$, ie $0 \neq 1$, the limit does not exist.

4. [10 points - 5 each] Calculate the following partial derivatives.

(a) Find f_x where $f(x, y) = \dfrac{x^2}{3x^2 - 5y^2}$

$$f_x(x,y) = \frac{2x(3x^2-5y^2) - 6x(x^2)}{(3x^2-5y^2)^2}$$

$$= \frac{6x^3 - 10xy^2 - 6x^3}{(3x^2-5y^2)^2} = \boxed{\frac{-10xy^2}{(3x^2-5y^2)^2}}$$

(b) Find g_{xy} where $g(x, y) = x\cos(y^2 + 2x^3)$

$$g_x = 1\cdot\cos(y^2+2x^3) + x\left[-\sin(y^2+2x^3)\cdot 6x^2\right]$$

$$g_x = \cos(y^2+2x^3) - 6x^3\sin(y^2+2x^3)$$

$$g_{xy} = -\sin(y^2+2x^3)\cdot 2y - 6x^3\cos(y^2+2x^3)\cdot 2y$$

$$g_{xy} = -2y\sin(y^2+2x^3) - 12x^3y\cos(y^2+2x^3)$$

[or start with

$$g_y = x(-\sin(y^2+2x^3)\cdot 2y) = -2xy\sin(y^2+2x^3)$$]

5. [5 points] Let $z = \sin(2x + y)$ where $x = s^2 - t^2$ and $y = s^2 + t^2$. Calculate $\frac{\partial z}{\partial s} = z_s$.

Note: Make sure your answer only has s's and t's in it.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$= \left[\cos(2x+y) \cdot 2\right]\left[2s\right] + \left[\cos(2x+y)\right]\left[2s\right]$$

$$= 4s\cos\left(2(s^2-t^2)+(s^2+t^2)\right)$$
$$+2s\cos\left(2(s^2-t^2)+(s^2+t^2)\right)$$

$$= 4s\cos\left(3s^2-t^2\right) + 2s\cos\left(3s^2-t^2\right)$$

$$= 6s\cos\left(3s^2-t^2\right)$$

6. [10 points - 5 each] Consider the function $f(x,y) = \frac{x}{x-y}$.

(a) Find the directional derivative of f at the point $(0,1)$ in the direction of $(4,1)$

(b) Find a vector that points in a direction of no change in the function at $(0,1)$.

(a) $\nabla f = \langle f_x, f_y \rangle$

$$\nabla f = \left\langle \frac{(1)(x-y)-(1)(x)}{(x-y)^2}, \frac{0(x-y)-(-1)(x)}{(x-y)^2} \right\rangle$$

$$\nabla f = \left\langle \frac{-y}{(x-y)^2}, \frac{x}{(x-y)^2} \right\rangle$$

$$\nabla f(0,1) = \left\langle \frac{-1}{(0-1)^2}, \frac{0}{(0-1)^2} \right\rangle = \langle -1, 0 \rangle$$

direction is (using $P=(0,1)$ and $Q=(4,1)$)

$\vec{PQ} = \langle 4-0, 1-1 \rangle = \langle 4, 0 \rangle$ $\quad |\vec{PQ}| = \sqrt{4^2+0^2} = 4$

Now make it a unit vector

$$\vec{u} = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{1}{4}\langle 4, 0 \rangle = \langle 1, 0 \rangle$$

$$D_{\vec{u}} f(0,1) = \nabla f(0,1) \cdot \vec{u} = \langle -1, 0 \rangle \langle 1, 0 \rangle = \boxed{-1}$$

(b) We need a vector that is perpendicular to $\nabla f(0,1) = \langle -1, 0 \rangle$. Use $\langle 0, 1 \rangle$.

Then $D_{\langle 0,1 \rangle} f(0,1) = \langle -1, 0 \rangle \cdot \langle 0, 1 \rangle = 0.$

Answer: $\boxed{\langle 0, 1 \rangle}$

7. [5 points] Find the linear approximation of $f(x,y) = 4\cos(2x - y)$ at the point $(\frac{\pi}{4}, 0)$. Then approximate $f(0.8, 0)$.

Recall that

$\cos(0) = 1$ $\cos(\pi/4) = \sqrt{2}/2$ $\cos(\pi/2) = 0$
$\sin(0) = 0$ $\sin(\pi/4) = \sqrt{2}/2$ $\sin(\pi/2) = 1$
$\pi \approx 3.14159$ $2\pi \approx 6.283185$ $\pi/4 \approx 0.7854$

$$f(\tfrac{\pi}{4}, 0) = 4\cos(2(\tfrac{\pi}{4}) - 0) = 4\cos(\tfrac{\pi}{2}) = 0$$

$$f_x = 4[-\sin(2x-y)\cdot 2] = -8\sin(2x-y)$$

$$f_y = 4[-\sin(2x-y)\cdot(-1)] = 4\sin(2x-y)$$

$$f_x(\tfrac{\pi}{4}, 0) = -8\sin(2(\tfrac{\pi}{4}) - 0) = -8\sin(\tfrac{\pi}{2}) = -8$$

$$f_y(\tfrac{\pi}{4}, 0) = 4\sin(2(\tfrac{\pi}{4}) - 0) = 4\sin(\tfrac{\pi}{2}) = 4$$

$$L(x,y) = 0 - 8(x - \tfrac{\pi}{4}) + 4(y - 0)$$

$$\boxed{L(x,y) = -8x + 4y + 2\pi}$$

$$f(0.8, 0) \approx L(0.8, 0) = -8(0.8) + 4(0) + 2\pi$$

$$\approx -6.4 + 6.28$$

$$\approx \boxed{-0.12}$$