# Math 4650 - Test 2 - Fall 2025

Name:______________________________________________

**<u>Directions</u>**:

Show steps for full credit.
Also so I can give you partial credit if needed.

| Score | | | |
|---|---|---|---|
| 1 | | 2 | |
| 3 | | A or B | |
| C or D | | | |
| Total | | | |

1. [10 points] Use partial fractions to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n} = 1$$

*In your derivation, give a formula for the $k$-th partial sum.*

2. [20 points - 10 each]

(a) Calculate the value of the series $\sum_{n=3}^{\infty} \left(\frac{2}{3}\right)^n$

(b) Determine if $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{3n+2}$ converges or diverges. Show why.

3. [10 points] Use the $\epsilon - \delta$ definition of limit to prove that $\lim_{x \to 2} x^2 = 4$

(A or B). [10 points] PICK ONE. If you do both, then I will grade A.

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A. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous at $a \in \mathbb{R}$. Prove that if $f(a) > 0$, then there exists $\delta > 0$ such that $f(x) > 0$ for all $x$ where $|x - a| < \delta$.

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B. Prove that if $f : D \to \mathbb{R}$ and $a$ is a limit point of $D$ with $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} f(x) = L_2$, then $L_1 = L_2$.

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(C or D). [10 points] PICK ONE. If you do both, then I will grade C.

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C. Let $f : D \to \mathbb{R}$ where $D \subseteq \mathbb{R}$ and $a \in D$. Prove: If $f$ is continuous at $a$, then $\lim_{n\to\infty} f(x_n) = f(a)$ for every sequence $(x_n)$ contained in $D$ with $x_n \to a$.

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D. Use the $\epsilon - \delta$ definition of continuity to prove that $f(x) = x^2 + x$ is continuous at any $a \in \mathbb{R}$.

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Extra page if you need it....