

Math 4650 - Test 1 - Fall 2025

Name: _____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.

Score			
1		2	
A or B		C or D	
E or F			
Total			

1. [20 points - 10 each] Find the supremum and infimum of each set if they exist.

First draw a picture of the set or list several elements of the set to get an idea of what's going on. Also, this will help me give you partial credit if your end result isn't fully correct.

(a) $Y = \left\{ 5 + \frac{(-1)^n}{n} \mid n = 1, 2, 3, 4, \dots \right\}$

(b) $Z = \{x^2 \mid x \in \mathbb{R} \text{ with } -1 \leq x\}$

2. [10 points] Prove that

$$\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2}$$

On this problem, you must prove the result using the ϵ -definition of limit. No theorems.

(A or B). [10 points] PICK ONE. If you do both, then I will grade A.

A. Let A and B be non-empty subsets of $\mathbb{R}$ that are both bounded from above. Prove: If $A \subseteq B$, then $\sup(A) \leq \sup(B)$.

B. Let A and B be non-empty subsets of $\mathbb{R}$. Suppose that the supremum of A and supremum of B exist. Prove: If $A \cap B$ is non-empty then $\sup(A \cap B) \leq \min\{\sup(A), \sup(B)\}$

(C or D). [10 points] PICK ONE. If you do both, then I will grade C.

C. Let a and b be real numbers. Prove that $|ab| = |a||b|$.

D. Let $c \in \mathbb{R}$ be a constant. Prove that $\lim_{n \rightarrow \infty} c = c$.

(E or F). [10 points] PICK ONE. If you do both, then I will grade E.

E. Suppose that (a_n) and (b_n) are sequences of real numbers such that $\lim_{n \rightarrow \infty} a_n = A$ and $\lim_{n \rightarrow \infty} b_n = B$ where A and B are real numbers. Let $\alpha \neq 0$ and $\beta \neq 0$ be real numbers. Prove that $\lim_{n \rightarrow \infty} \alpha a_n + \beta b_n = \alpha A + \beta B$.

F. (Squeeze Theorem) Suppose that (a_n) , (b_n) , and (c_n) are sequences of real numbers such that $a_n \leq b_n \leq c_n$ for all n . If both (a_n) and (c_n) both converge to L , then (b_n) converges to L .

Extra page if you need it....