Math 4550 - Test 2 - Fall 2025

Name:____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.

Score			
1		2	
3		4	
A or B		C or D	
Total			

1. [10 points - 5 each] Let $\phi : \mathbb{Z} \to \mathbb{Z}$ be a homomorphism. Suppose that $\phi(1) = 4$.

(a) Calculate $\phi(3)$.

(b) Calculate $\phi(-1)$

---

2. [10 points - 5 each] Let $\phi : \mathbb{Z} \to \mathbb{Z}$ be defined by $\phi(n) = -2n$.

(a) Prove that ϕ is a homomorphism.

(b) What is the kernel of ϕ ?

3. [10 points] Find all homomorphisms $\phi : \mathbb{Z}_3 \to \mathbb{Z}_6$

4. [12 points - 3 each] Let $G = \mathbb{Z}_8$ and $H = \langle \overline{4} \rangle = \{\overline{0}, \overline{4}\}$.
H is a normal subgroup of G. You don't have to check this fact.

(a) List the elements of G/H. How many elements are in G/H ?

(b) Find the inverse of $\overline{3} + H$ in G/H.

(c) Find the order of $\overline{2} + H$ in G/H.

(d) Is G/H cyclic? If G/H is cyclic, state a generator.

(A or B). [10 points] **Pick ONE of the following proofs.**
Only pick one; if you do both then I will grade A.

A. Let G_1 and G_2 be groups. Let $\phi : G_1 \to G_2$ be a homomorphism. Prove that $\ker(\phi)$ is a subgroup of G_1.

B. Let G be a group with identity element e. Suppose that $|G| = n$. Prove that $x^n = e$ for all $x \in G$.

(C or D). [10 points] **Pick <u>ONE</u> of the following proofs.**
Only pick one; if you do both then I will grade C.

C. Let G be a group and H be a normal subgroup of G. Prove that if G is cyclic, then G/H is cyclic.

D. Let G_1 and G_2 be groups. Let e_1 and e_2 be the identity elements of G_1 and G_2 respectively. Let $\phi : G_1 \to G_2$ be a homomorphism. Prove that if $\ker(\phi) = \{e_1\}$, then ϕ is one-to-one.

Extra page if you need it....