

Math 4550 - Test 1 - Fall 2025

Name:_____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.

Score			
1		2	
3		A or B	
C or D			
Total			

1. [15 points - 5 each]

(a) Draw a picture of U_8 . Let $\zeta = e^{2\pi i/8}$. Make sure to label all the points.

(b) Find the inverse of ζ^3 .

(c) Is $H = \{1, \zeta^2, \zeta^4\}$ a subgroup of U_8 ? Why or why not.

2. [15 points - 5 each] Consider the group $D_8 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$.

(a) Simplify the product $r^{-3}sr^5sr^4$ into one of the elements from D_8 listed above.

(b) Calculate the order of sr^3 .

(c) Calculate the subgroup generated by r^2 .

3. [15 points - 5 each]

(a) List the elements of $\mathbb{Z}_3 \times \mathbb{Z}_3$

(b) Calculate the order of $(\bar{1}, \bar{2})$.

(c) Find the inverse of $(\bar{1}, \bar{1})$

(A or B). [10 points] **Pick ONE of the following proofs.**
Only pick one; if you do both then I will grade A.

A. Show that

$$H = \{2x + 3y \mid x, y \in \mathbb{Z}\}$$

is a subgroup of $\mathbb{Z}$.

B. Let G be a group where every element is its own inverse. Prove that G is abelian.

(C or D). [10 points] **Pick ONE of the following proofs.**

Only pick one; if you do both then I will grade C.

C. Let $\mathbb{R}^* = \mathbb{R} - \{0\}$. Given $a, b \in \mathbb{R}^*$ define $a * b = |ab|$. Prove that $\mathbb{R}^*$ is not a group under this operation.

*Example computation with $a = -1, b = 0.5$: $(-1) * (0.5) = |(-1)(0.5)| = 0.5$.*

D. Let G be a group. Prove that if $G \times G$ is cyclic, then G is cyclic.

Extra page if you need it....