Math 3450 - Spring 2024 - Test 1

Name:____

Recall that

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

Score			
1		2	
3		4	
5		6	
7		Total	

1. [8 points] List all of the elements from the set

$$A = \{2s - 3t \mid s, t \in \mathbb{Z} \text{ and } 1 \leq s \leq 2 \text{ and } -1 \leq t \leq 0\}$$

2. [20 points - 4 each] Let $A = \{5, 2, 10, -1, 4, 8, 0\}$, $B = \{5, 10, -2, \frac{4}{5}, 11, 0\}$, $C = \{10, 11, 1\}$, $D = \{10, 2\}$, $E = \{0, 1, 2\}$. Compute the following.

(a) $A \cap C$

(b) $B - C$.

(c) $C \cup D$

(d) $C \times D$

(e) The power set $\mathcal{P}(E)$.

3. [10 points - 5 each] Explain why each is true or false.

(a) True or False? $-2 \equiv 28 \pmod 6$

(b) True or False? $\overline{1} = \overline{2045}$ in $\mathbb{Z}_4$

4. [10 points] Let $A_n = \{x \in \mathbb{Z} \mid -n \leq x \leq n\}$.

(a) List the elements in the sets A_1, A_2, A_3, and A_4.

(b) Calculate $\bigcap_{n=2}^{\infty} A_n$ and $\bigcup_{n=5}^{\infty} A_n$.

5. [10 points]

Pick ONE of the following to prove. Only pick one. If you do both then I will grade (A).

A) Let A, B, and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

B. Let A, B, C be sets. Prove that $(A \cap B) - C = (A - C) \cap (B - C)$.

6. [10 points]

Pick ONE of the following to prove. Only pick one. If you do both then I will grade (C).

C) Let $S = \mathbb{Z} \times (\mathbb{Z} - \{0\})$. Define the relation $\sim$ on S where $(a, b) \sim (c, d)$ if and only if $ad = bc$. Prove that $\sim$ is an equivalence relation.

D. Consider the set of integers $\mathbb{Z}$. Let $n \in \mathbb{Z}$ with $n \geq 2$. Given $a, b \in \mathbb{Z}$ define $a \sim b$ to mean that n divides $a - b$. Prove that $\sim$ is an equivalence relation.

7. [10 points]

Pick ONE of the following to prove. Only pick one. If you do both then I will grade (E).

E. Let A and B bet sets. Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

F. Prove that $\{9^n \mid n \in \mathbb{N}\} \subseteq \{3^n \mid n \in \mathbb{N}\}$ but $\{9^n \mid n \in \mathbb{N}\} \neq \{3^n \mid n \in \mathbb{N}\}$.

Extra space if you need it.