Weds. 1

HW 2

(4) Let A and B be sets.

Prove: A=B iff A-B=\$

Need to show

ed to show

(=>) If A \le B + then A - B = \(\begin{array}{c} \frac{\text{pf.}}{\text{Show } \text{Q leads}} \\ \text{to a contradiction} \end{array}

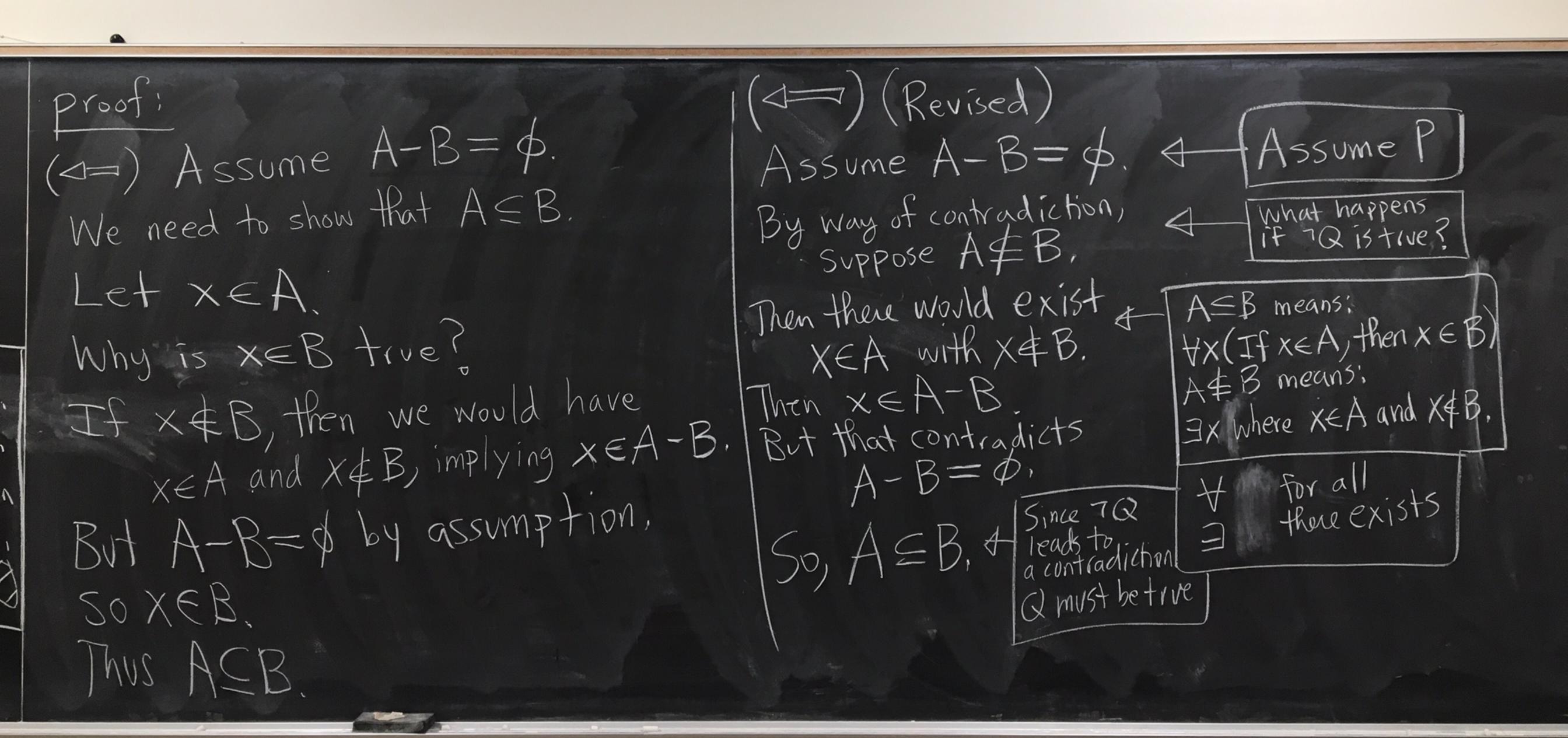
(4) If A-B= & then A < B

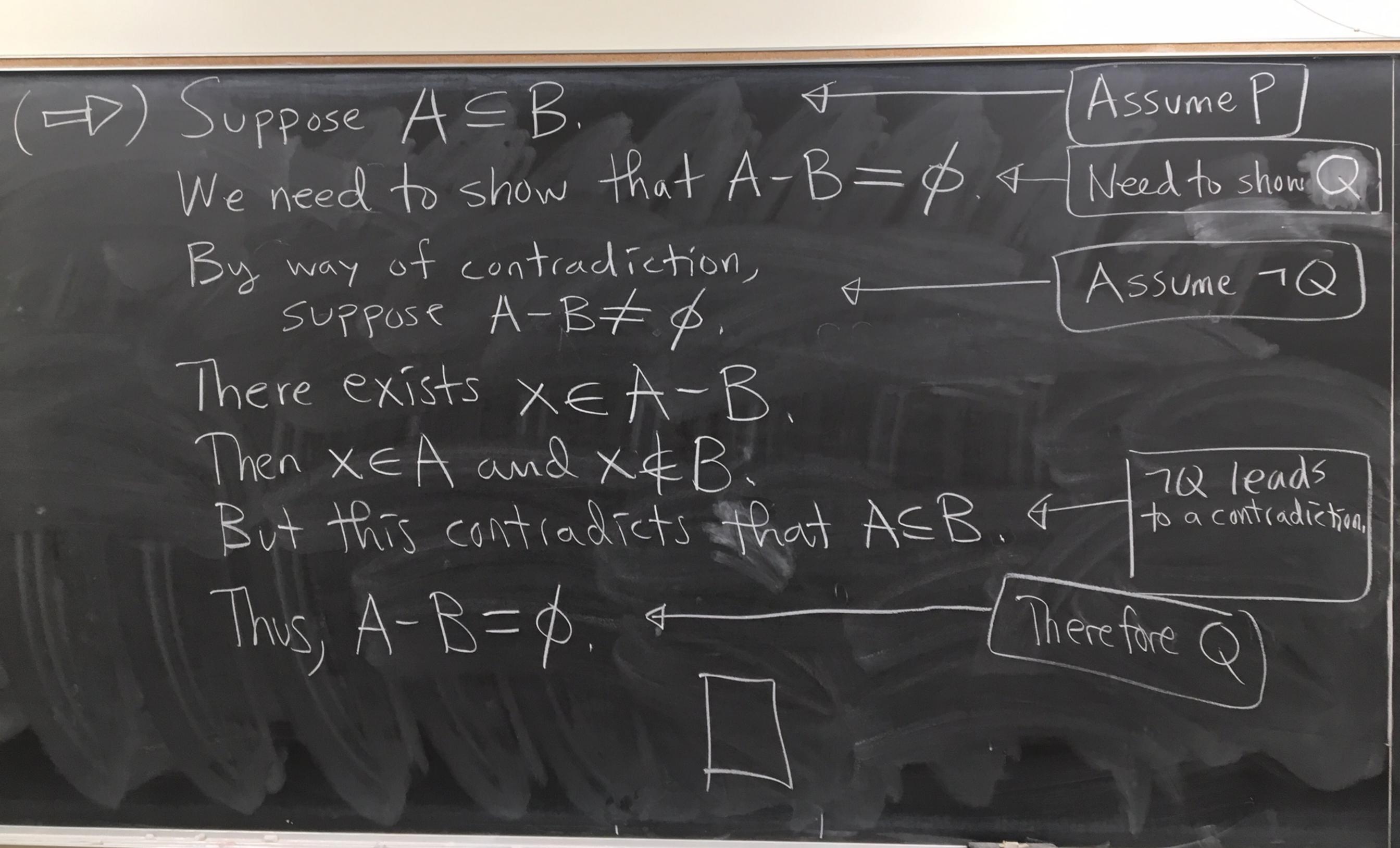
If P, then Q

Contradiction

If P, then Q.

Thus, Qistive,





b-ax=17-5X Division Alg. Continued b=aq+1 17=5.3+2 not in S a=5, b=175 EXO Define $S = \{b - \alpha \times | x \in \mathbb{Z} \text{ and } \}$ $S = \{b - \alpha \times | b - \alpha \times \geq 0\}$ $= \begin{cases} |x \in \mathbb{Z} \text{ and } \\ |7-5x > 0 \end{cases}$ {2,7,12,17,22,000} Smallest integer in 51

Division Algorithm Let a, bell with a 70. Then there exists unique integers q and r with b=aqtr and 0<r< a. Proof? Setting X=-1 gives (existence) Let S=\{\int b-ax\}\)
\[
\text{ond}\]
\[
\text{ond}\] b-ax=b+a70(because $b \ge 0$ & a > 0) So, bta is a positive integer in) Let's show that Stop. cases; Suppose 6<0. Setting X=Zb gives b-ax = b-a(2b) = b(1-2a) > 0caseli Suppose b>0. Since b<0 and a>l gives 1-2a<0)

So, $b-a(2b) \in S$. By case I and case 2, 5 + p. Since S is not empty and S is a set of non-negative integers, S must have a smallest element. Let r be the smallest element in S. So there exists qEZ with r=b-aq and r=b-aq7,0,

(So, b = aq + r with $0 \le r$. We now show r < a. Suppose that rza. Then r-a>0. r-a=(b-aq)-a=b-a(q+1).Since $r-a \ge 0$ and r-a=b-a(q+1)we know $r-a \in S.$ But r-a<r-since a>0. element of 5 that is smaller than r. But then r-a would be an

This contradicts the assumption that cis the smallest element in S. Thus, there exists 9, r = 2 with b=agtr and OEr < a. (uniqueness) Suppose we have b=aq+r and b= aq'+r' with q,r,q,r'= 2 and 0 < r < a and 0<r<a We will show that 9=9 and r=r'

