Weds, 8/28

<u>Def:</u> We say that two sets A and B are <u>disjoint</u> if $A \cap B = \phi$.

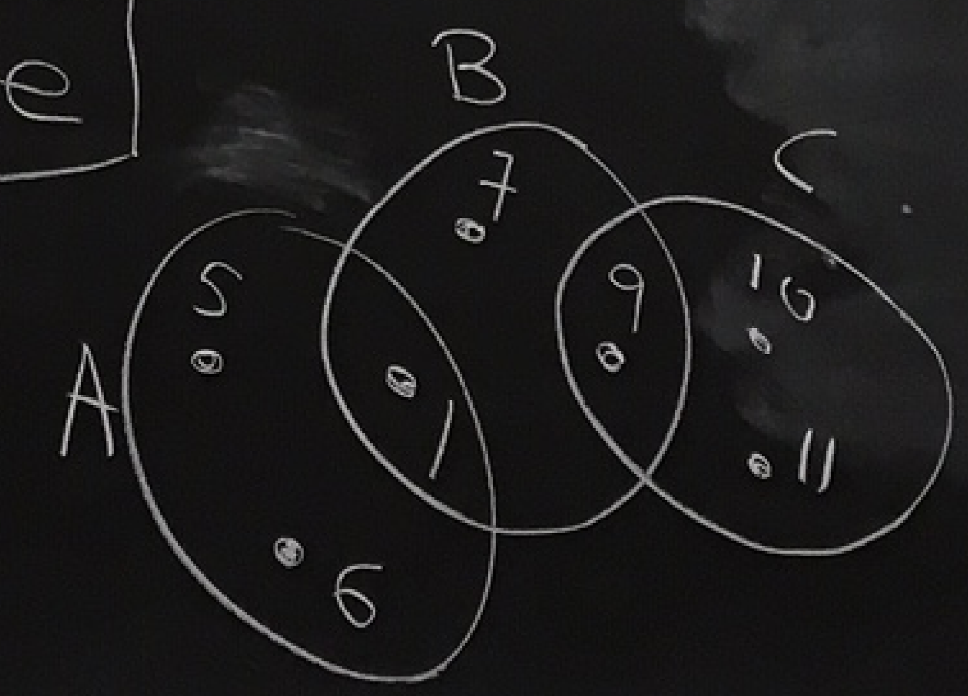
<u>Ex:</u> $A = \{1, 2, 4\}$
$B = \{7, 3\}$

$A \cap B = \phi$
so A and B are disjoint.

<u>HW 2</u>

7 Let A, B, C be sets.
<u>Prove or disprove:</u>
If $A \cap B \neq \phi$ and $B \cap C \neq \phi$ then $A \cap C \neq \phi$.

$\boxed{\text{False}}$



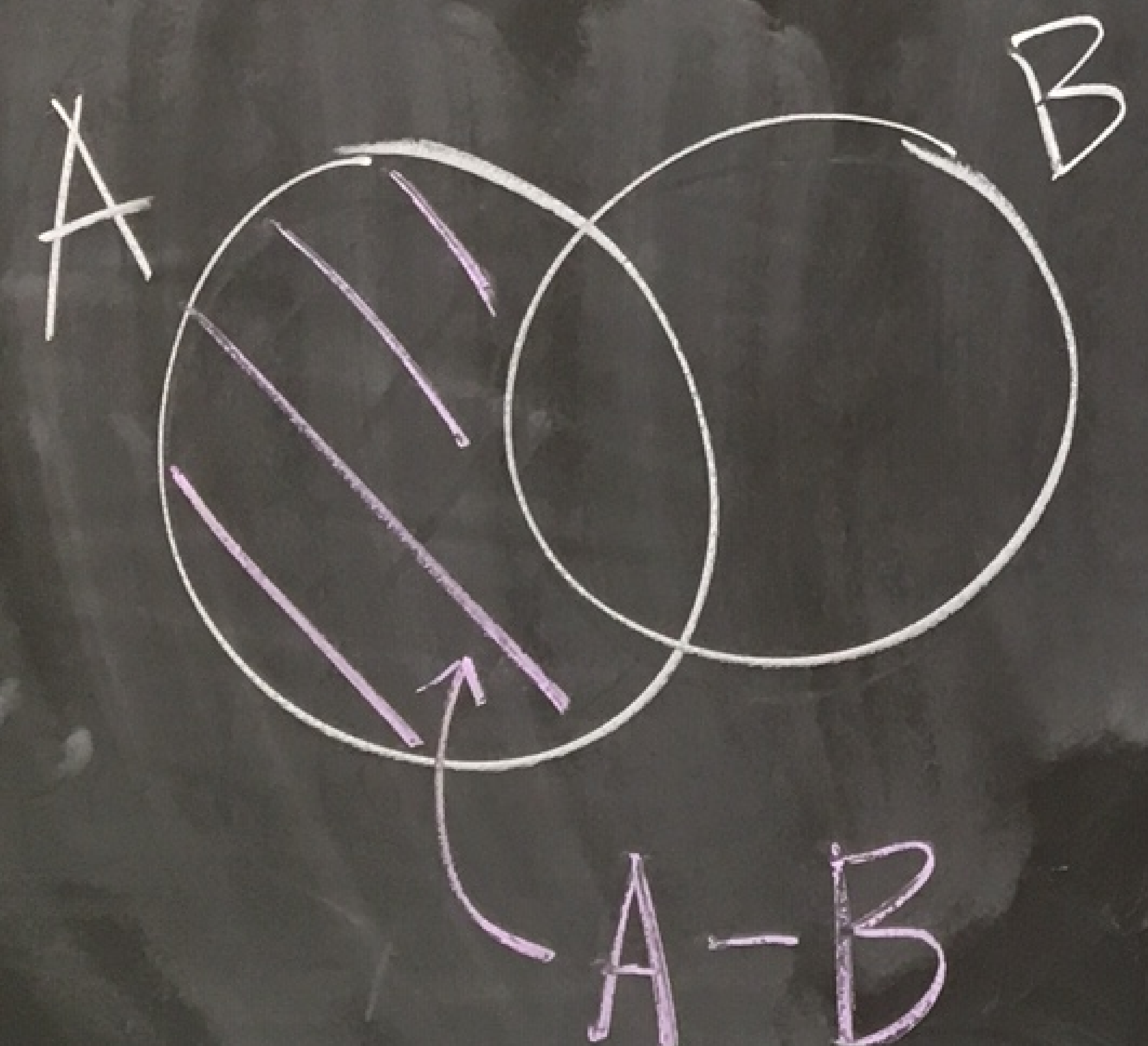
$A = \{1, 5, 6\}$
$B = \{1, 7, 9\}$
$C = \{9, 10, 11\}$

$A \cap B = \{1\} \neq \phi$
$B \cap C = \{9\} \neq \phi$
but
$A \cap C = \phi$.

This is called a counterexample.

Def: Let A and B be sets.
The **difference** of A and B is

$$A \setminus B = A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



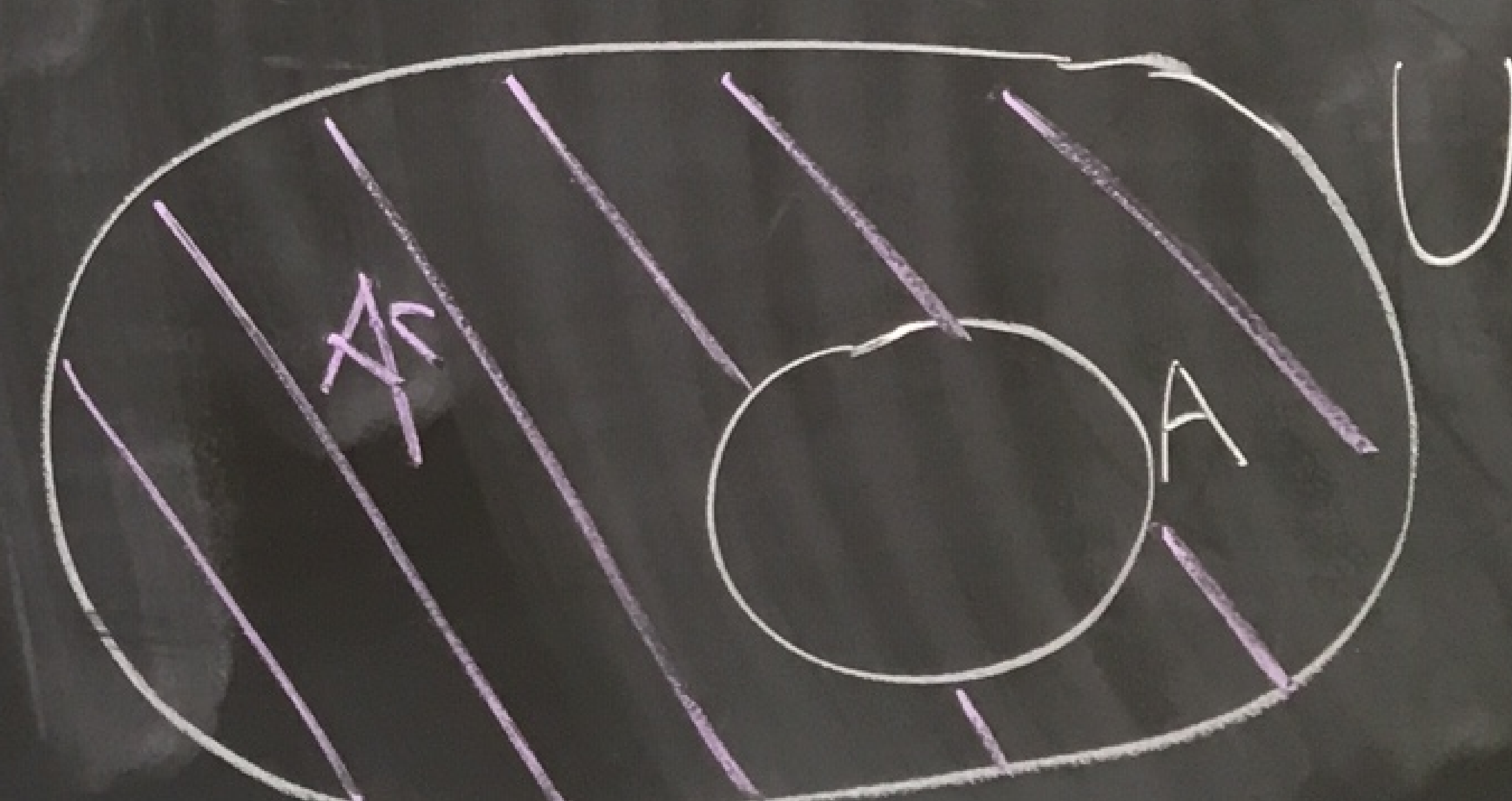
Ex: $A = \{\sqrt{2}, 4, 7, \frac{11}{3}, \pi^2\}$

$B = \{5, 7, \pi, 10, \frac{11}{3}\}$

$A - B = \{\sqrt{2}, 4, \pi^2\}$

Sometimes we have a "universal set" or "universe" that all our sets live in.

<u>Def:</u> Let A be a set where U is a universal set. The <u>complement</u> of A, denoted by $\overline{A}$ or A^c or A', is the set $A^c = U - A$.



Theorem: (de Morgan's laws)

Let A and B be sets where U is a universal set. Then

① $(A \cup B)^c = A^c \cap B^c$

② $(A \cap B)^c = A^c \cup B^c$

proof: ① ($\Rightarrow$) First let's show $(A \cup B)^c \subseteq A^c \cap B^c$.

Let $x \in (A \cup B)^c$.

So $x \in U - (A \cup B)$.

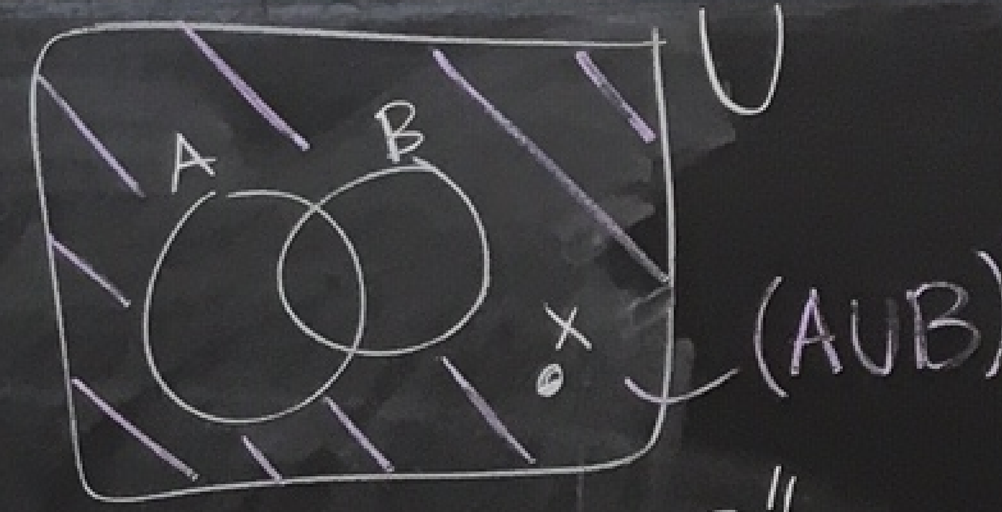
Thus $x \notin A \cup B$.

So the following is not true: $\underbrace{\text{"}x \in A \text{ or } x \in B\text{"}}_{\text{means } x \in A \cup B}$

So, $x \notin A$ and $x \notin B$.

Thus, $x \in A^c$ and $x \in B^c$.

Ergo, $x \in A^c \cap B^c$.



$$\boxed{\neg(P \text{ or } Q) \text{ is } (\neg P) \text{ and } (\neg Q)}$$

($\Leftarrow$) Now we show that $A^c \cap B^c \subseteq (A \cup B)^c$.

Let $x \in A^c \cap B^c$.

Then $x \in A^c$ and $x \in B^c$

So, $x \notin A$ and $x \notin B$.

Hence $x \notin A \cup B$.

Consequently $x \in (A \cup B)^c$.

By ($\Rightarrow$) and ($\Leftarrow$) we have $(A \cup B)^c = A^c \cap B^c$ $\blacksquare$

Simpler:

$x \in (A \cup B)^c$

iff $x \notin A \cup B$

iff $x \notin A$ and $x \notin B$

iff $x \in A^c$ and $x \in B^c$

iff $x \in A^c \cap B^c$.

So, $(A \cup B)^c = A^c \cap B^c$.

Def: Let A and B be two sets. The Cartesian product (or cross product) of A and B is

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Ex: $A = \{1, -1, e\}$

$B = \{0, e\}$

$A \times B = \{(1,0), (1,e), (-1,0), (-1,e), (e,0), (e,e)\}$

For ordered pairs by definition

$$(x,y) = (w,z)$$

if and only if

$$x = w \text{ and } y = z.$$

es - Wikipedia × W Ordered pair - Wikipedia × +

en.wikipedia.org/wiki/Ordered_pair#Informal_and_formal_definitions

Wiener's definition [edit]

Norbert Wiener proposed the first set theoretical definition of the ordered pair in 1914:[6]

$$(a, b) := \{\{\{a\}, \emptyset\}, \{\{b\}\}\}.$$

He observed that this definition made it possible to define the types of *Principia Mathematica* as sets. *Principia Mathematica* had taken types, and hence relations of all arities, as primitive.

Wiener used {{*b*}} instead of {*b*} to make the definition compatible with type theory where all elements in a class must be of the same "type". With *b* nested within an additional set, its type is equal to $\{\{a\}, \emptyset\}$'s.

Hausdorff's definition [edit]

About the same time as Wiener (1914), Felix Hausdorff proposed his definition:

$$(a, b) := \{\{a, 1\}, \{b, 2\}\}$$

"where 1 and 2 are two distinct objects different from a and b."[7]

Kuratowski's definition [edit]

In 1921 Kazimierz Kuratowski offered the now-accepted definition[8][9] of the ordered pair (*a*, *b*):

$$(a,\ b)_K\ :=\ \{\{a\},\ \{a,\ b\}\}.$$

Note that this definition is used even when the first and the second coordinates are identical:

$$(x,\ x)_K = \{\{x\}, \{x,\ x\}\} = \{\{x\},\ \{x\}\} = \{\{x\}\}$$

Given some ordered pair *p*, the property "*x* is the first coordinate of *p*" can be formulated as:

$$\forall Y \in p : x \in Y.$$

The property "*x* is the second coordinate of *p*" can be formulated as:

$$(\exists Y \in p : x \in Y) \land (\forall Y_1, Y_2 \in p : Y_1 \neq Y_2 \rightarrow (x \notin Y_1 \lor x \notin Y_2)).$$

In the case that the left and right coordinates are identical, the right conjunct

to search

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Def: Let A be a set.

We define the power set of A to be the set of all subsets of A, that is

$$\mathcal{P}(A) = \{ B \mid B \subseteq A \}$$

Ex: $A = \{1, 2\}$

subsets of A
ϕ
$\{1\}$
$\{2\}$
$\{1,2\}$

$\mathcal{P}(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

Note: $\phi \subseteq A$ for any set A

Def: $X \subseteq Y$ means

$(\forall x)(\text{If } x \in X, \text{ then } x \in Y)$

$(\forall x)\underbrace{(\text{If } \underbrace{x \in \phi}_{F}, \text{ then } x \in A)}_{T}$ ← True