Set Theory Review

natural numbers → $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

integers → $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

rational numbers → $\mathbb{Q} = \left\{ \frac{x}{y} \;\middle|\; \begin{matrix} x, y \in \mathbb{Z} \\ y \neq 0 \end{matrix} \right\}$

{ description of elements | condition on elements }

Notation: Let A be a set. If x is in A we write $x \in A$. Otherwise we write $x \notin A$.

real numbers → $\mathbb{R} = \{ x \mid x \text{ has a decimal expansion} \}$

$= \{ \sqrt{3} \approx 1.732,\ 1 = 1.0,\ \frac{1}{2} = 0.5, \ldots \}$

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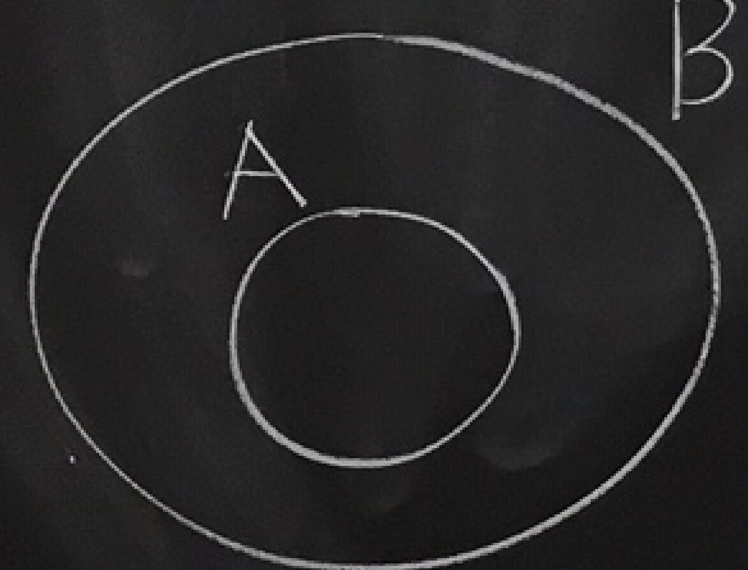
Def: Let A and B be sets.
We say that A is a subset of B if every element of A is also an element of B.

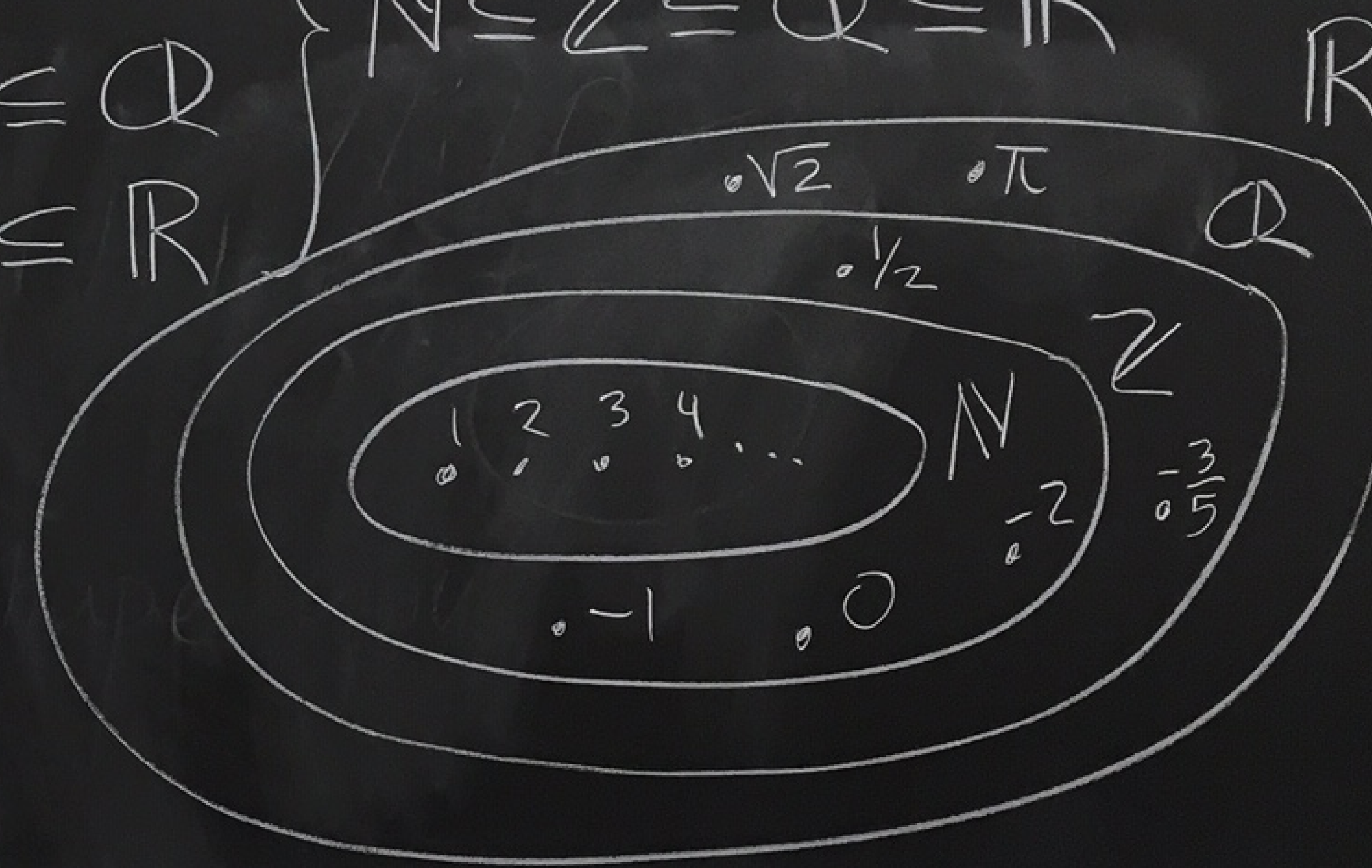


If A is a subset of B then we write
$$A \subseteq B.$$
(Some people just write $A \subset B$.)

Ex:

$\mathbb{N} \subseteq \mathbb{Z}$
$\mathbb{Z} \subseteq \mathbb{Q}$
$\mathbb{Q} \subseteq \mathbb{R}$

$\Big\}\ \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$



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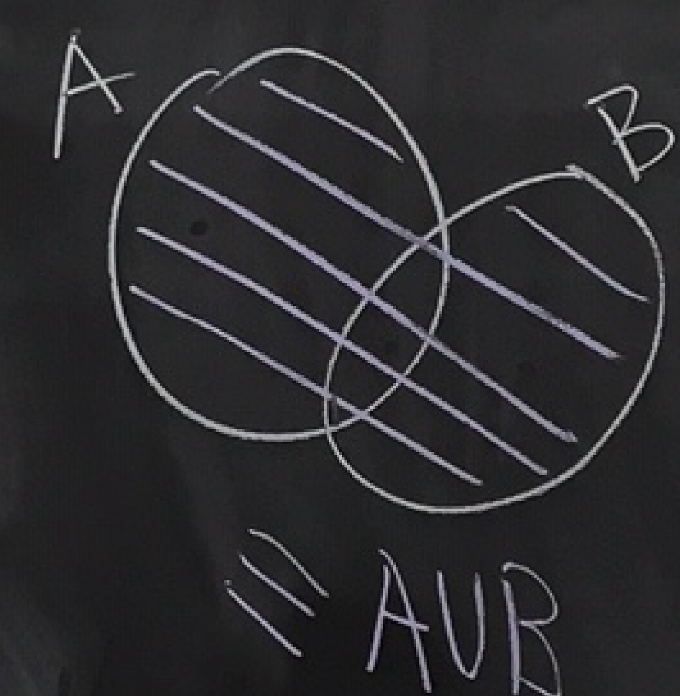
$, \frac{1}{2} = 0.5, \dots \}$

Method to show $A=B$ when A and B are sets

1. Show $A \subseteq B$
2. Show $B \subseteq A$

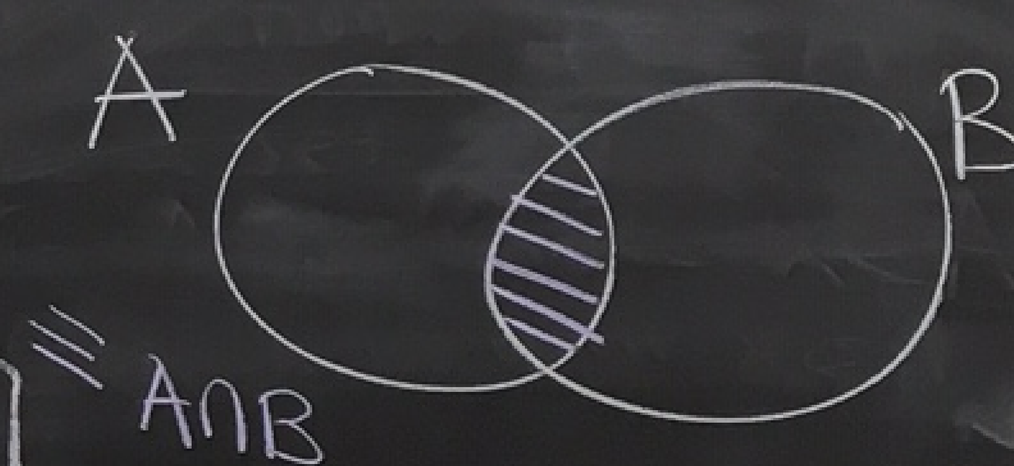
Def: Let A and B be sets. The union of A and B is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



The intersection of A and B is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



P	Q	P or Q	P and Q	If P, then Q
T	F	T	F	F
T	T	T	T	T
F	T	T	F	T
F	F	F	F	T

Ren Q

<u>Ex:</u> $A = \{1, 5, 7, 13, -\frac{1}{2}, \sqrt{2}\}$

$B = \{\sqrt{2}, 0, 52, -3.5, 7\}$

"$\underbrace{7 \in A}_{T}$ or $\underbrace{7 \in B}_{T}$" is True

So, $7 \in A \cup B$

$A \cup B = \{1, 5, 7, 13, -\frac{1}{2}, \sqrt{2}, 0, 52, -3.5\}$

$A \cap B = \{\sqrt{2}, 7\}$

"$\underbrace{0 \in A}_{T}$ or $\underbrace{0 \in B}_{F}$" is True

So, $0 \in A \cup B$

"$\underbrace{\sqrt{2} \in A}_{T}$ and $\underbrace{\sqrt{2} \in B}_{T}$" is True

So, $\sqrt{2} \in A \cap B$

$C = \{1, 2, 3, 4, 5$
$D = \{3, 5, 7\}$
$E = \{2, 1, -6\}$

Ex: Let C, D, E be sets.
Prove that $C \cap (D \cup E) = (C \cap D) \cup (C \cap E)$

proof:

$\boxed{\subseteq}$ We first show that $C \cap (D \cup E) \subseteq (C \cap D) \cup (C \cap E)$.

Let $x \in C \cap (D \cup E)$.

So, $x \in C$ and $x \in D \cup E$.

Thus, $x \in C$ and $(x \in D$ or $x \in E)$

$C = \{1, 2, 3, 4, 5\}$

$D = \{3, 5, 7\}$

$E = \{2, 1, -6\}$

$\cap D) \cup (C \cap E)$.

$D \cup E = \{1, 2, 3, 5, 7, -6\}$

$C \cap (D \cup E) = \{1, 2, 3, 5\}$

$C \cap D = \{3, 5\}$ $\quad C \cap E = \{1, 2\}$

$(C \cap D) \cup (C \cap E) = \{1, 2, 3, 5\}$

Hooray! They're equal

So either
① $x \in C$ and $x \in D$
or ② $x \in C$ and $x \in E$

case 1: Suppose $x \in C$ and $x \in D$.
Then $x \in C \cap D$.
Hence $x \in (C \cap D) \cup (C \cap E)$.

case 2: Suppose $x \in C$ and $x \in E$.
Then, $x \in C \cap E$.
Ergo, $x \in (C \cap D) \cup (C \cap E)$.

So, in either case
$x \in (C \cap D) \cup (C \cap E)$.
Thus, if $x \in C \cap (D \cup E)$
then $x \in (C \cap D) \cup (C \cap E)$.
So,
$C \cap (D \cup E) \subseteq (C \cap D) \cup (C \cap E)$.

$\boxed{\supseteq}$ Let's now show $(C \cap D) \cup (C \cap E) \subseteq C \cap (D \cup E)$

Let $w \in (C \cap D) \cup (C \cap E)$

So, $w \in C \cap D$ or $w \in C \cap E$.

<u>case 1:</u> Suppose $w \in C \cap D$.

Then $w \in C$ and $w \in D$.

Since $w \in D$ we know $w \in D \cup E$.

It follows that $w \in C$ and $w \in D \cup E$.

Consequently, $w \in C \cap (D \cup E)$.

<u>case 2:</u> Suppose $w \in C \cap E$.

Then $w \in C$ and $w \in E$.

Since $w \in E$ we know $w \in D \cup E$.

Since $w \in C$ and $w \in D \cup E$

we know $w \in C \cap (D \cup E)$.

$\cap E$.

E.

$\in D \cup E$.

$)$.

We have show that if
$w \in (C \cap D) \cup (C \cap E)$
then we must have
$w \in C \cap (D \cup E)$.

So,
$(C \cap D) \cup (C \cap E) \subseteq C \cap (D \cup E)$.

Therefore by parts $\boxed{\subseteq}$ and $\boxed{\supseteq}$ we know

$$(C \cap D) \cup (C \cap E) = C \cap (D \cup E).$$

$\square$