Nov 4
Monday

Last time

① $f(W \cap Z) \subseteq f(W) \cap f(Z)$

Hammock 12.6 #7

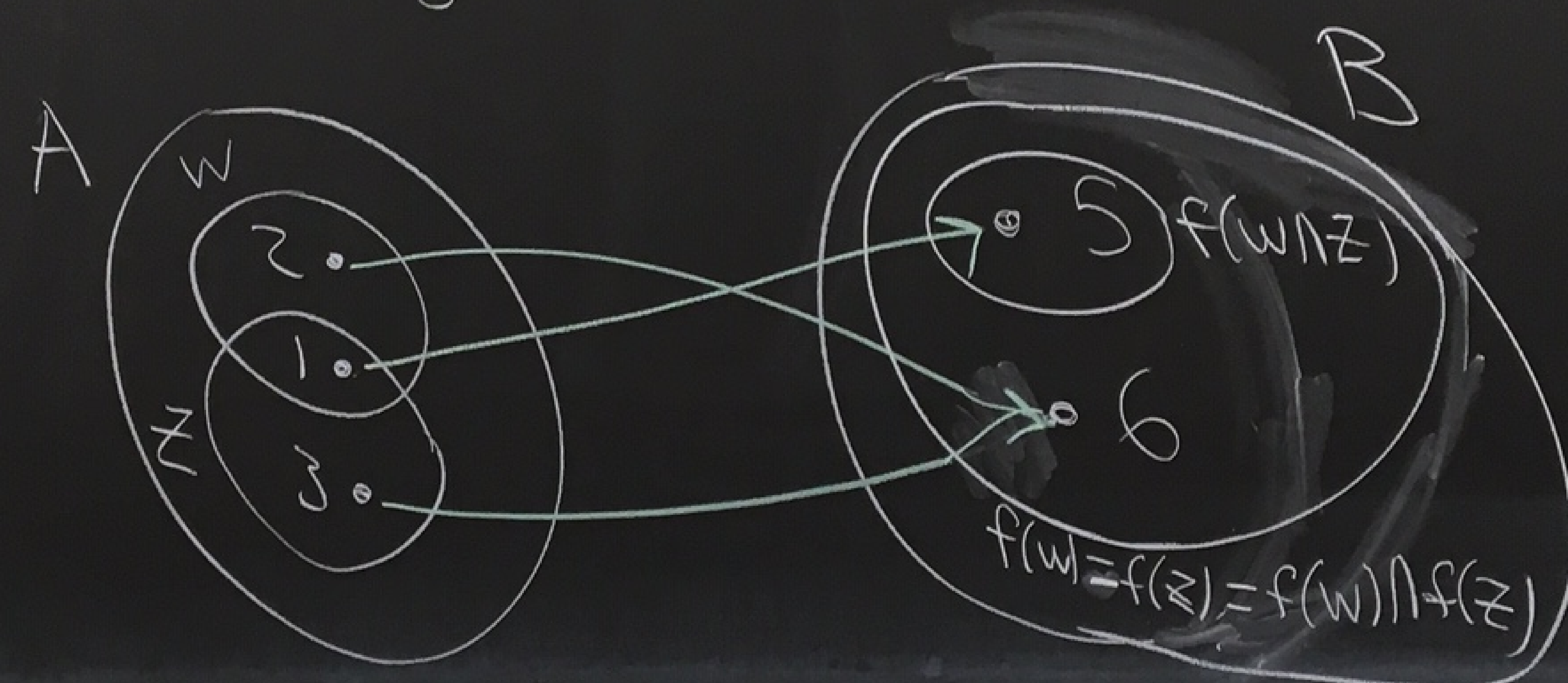
Continued from last time ...

Hammock 12.6 #8

② $f: A \to B$

$W \subseteq A$ and $Z \subseteq A$

Show $f(W \cap Z) = f(W) \cap f(Z)$ might not be true.



$W = \{1, 2\}$

$Z = \{1, 3\}$

$f(W) = \{5, 6\}$

$f(Z) = \{5, 6\}$

$f(W \cap Z) = \{5\}$

$f(W) \cap f(Z) = \{5, 6\}$

③ $f(W \cup Z) = f(W) \cup f(Z)$

HW 4 #14

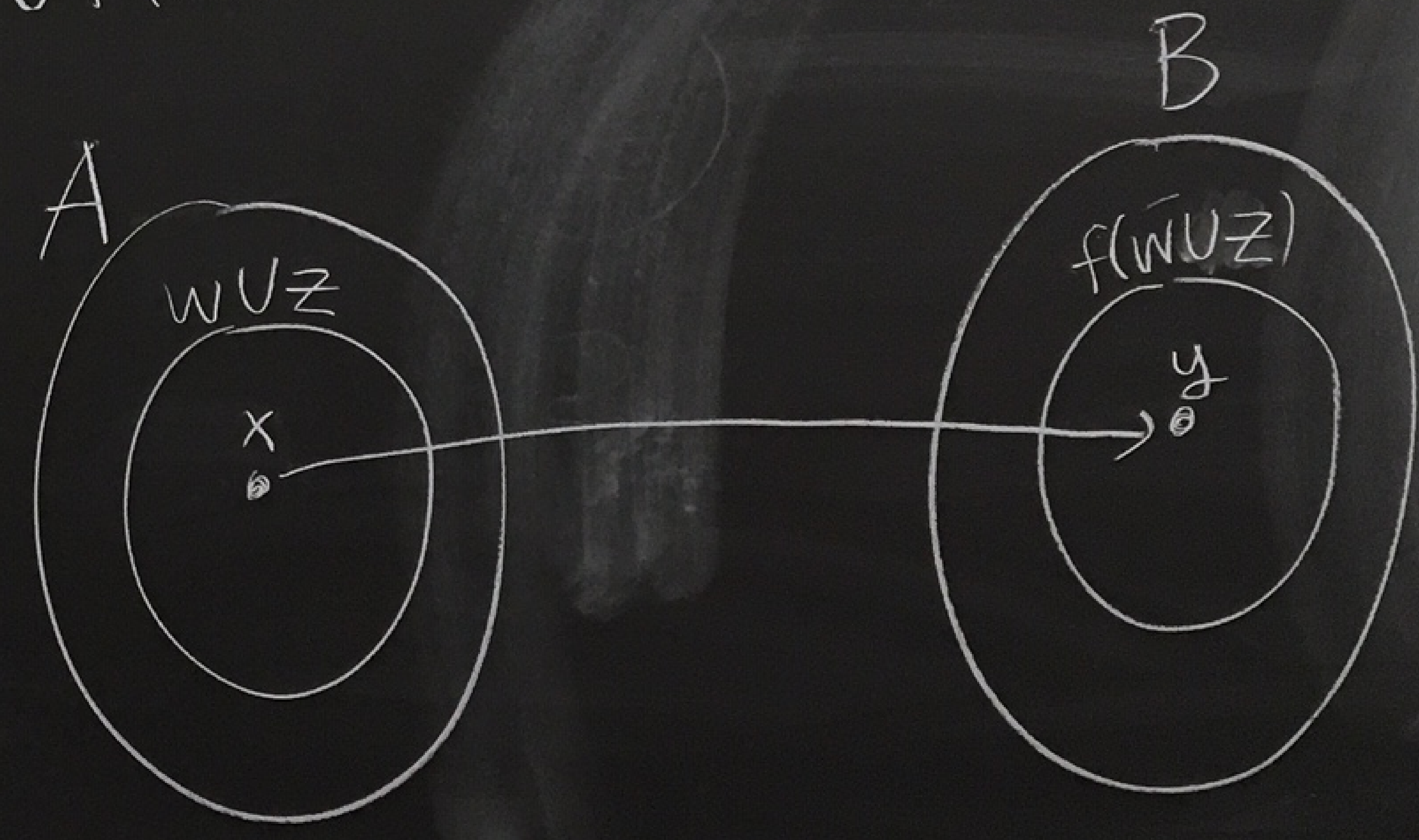
<u>proof</u>:

$\boxed{\subseteq}$: We will show $f(W \cup Z) \subseteq f(W) \cup f(Z)$.

Let $y \in f(W \cup Z)$.

So there exists $x \in W \cup Z$ with $f(x) = y$.

Note this implies $x \in W$ or $x \in Z$.

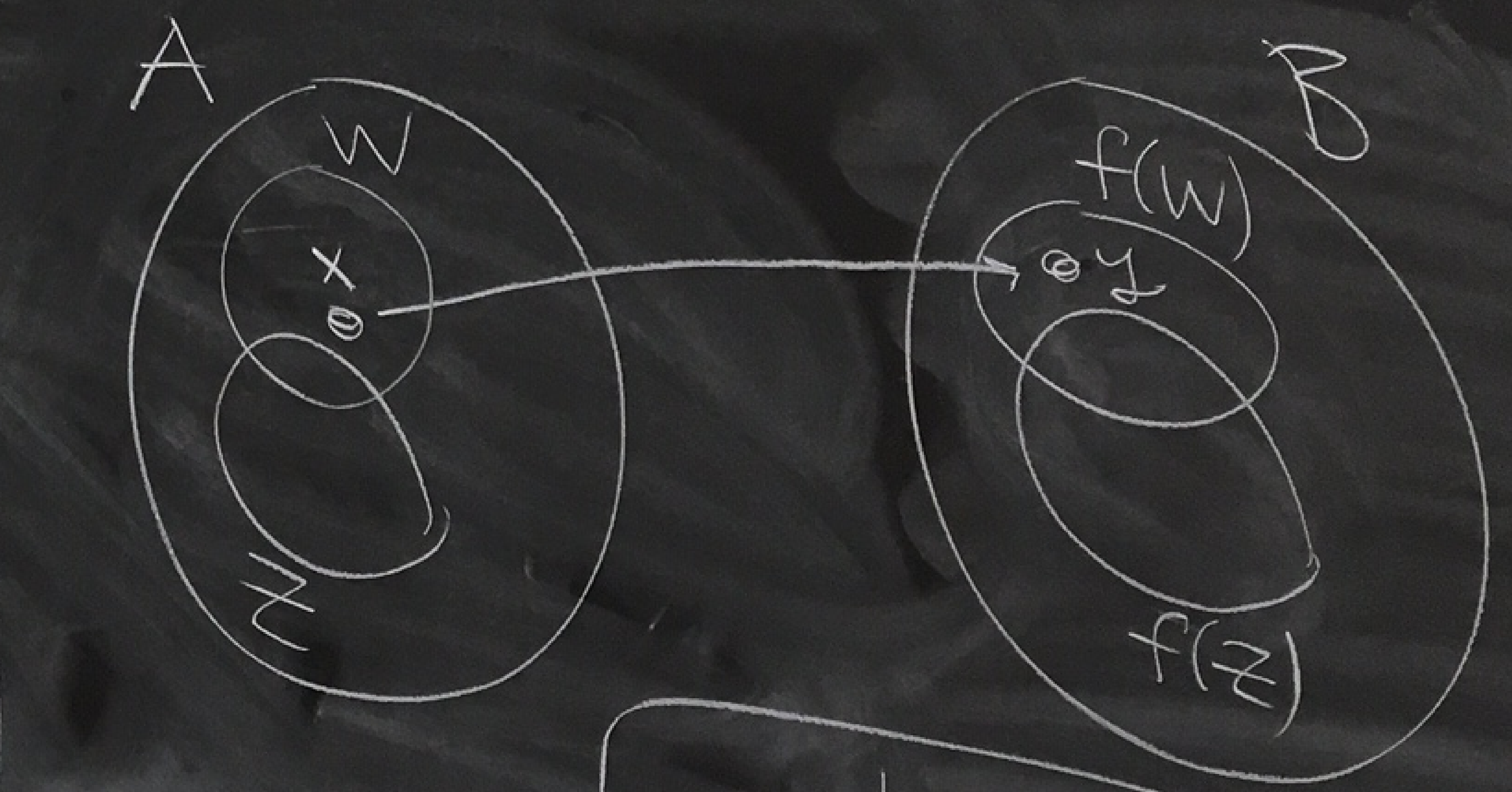


case 1: If $x \in W$, then $y = f(x)$ is in $f(W)$.
case 2: If $x \in Z$, then $y = f(x)$ is in $f(Z)$.

Thus, $y \in f(W)$ or $y \in f(Z)$.

So, $y \in f(W) \cup f(Z)$. Therefore, $f(W \cup Z) \subseteq f(W) \cup f(Z)$.

$\boxed{\supseteq}$: We will show
$f(W) \cup f(Z) \subseteq f(W \cup Z)$.
Let $y \in f(W) \cup f(Z)$.
So, $y \in f(W)$ or $y \in f(Z)$.



case 1 picture

Case 1: Suppose $y \in f(W)$.

Then there exists $x \in W$ with $y = f(x)$.

Since $x \in W$ we know $x \in W \cup Z$.

Since $x \in W \cup Z$ and $y = f(x)$ we know $y \in f(W \cup Z)$.

Case 2: Suppose $y \in f(Z)$.

Then there exists $x \in Z$ with $y = f(x)$.

Since $x \in Z$ we know $x \in W \cup Z$.

Since $x \in W \cup Z$ and $y = f(x)$ we know $y \in f(W \cup Z)$.

In either case, $y \in f(W \cup Z)$.

Therefore, $f(W) \cup f(Z) \subseteq f(W \cup Z)$. $\blacksquare$

HW 4

(14) $f : A \to B$

(b) If $W \subseteq B$ and $Z \subseteq B$
then $f^{-1}(W \cap Z) = f^{-1}(W) \cap f^{-1}(Z)$.

(c) If $Y \subseteq B$, then

$$A - f^{-1}(Y) \subseteq f^{-1}(B - Y)$$

proof of (b)

$\boxed{\subseteq}$: We will show $f^{-1}(W \cap Z) \subseteq f^{-1}(W) \cap f^{-1}(Z)$

Let $a \in f^{-1}(W \cap Z)$.

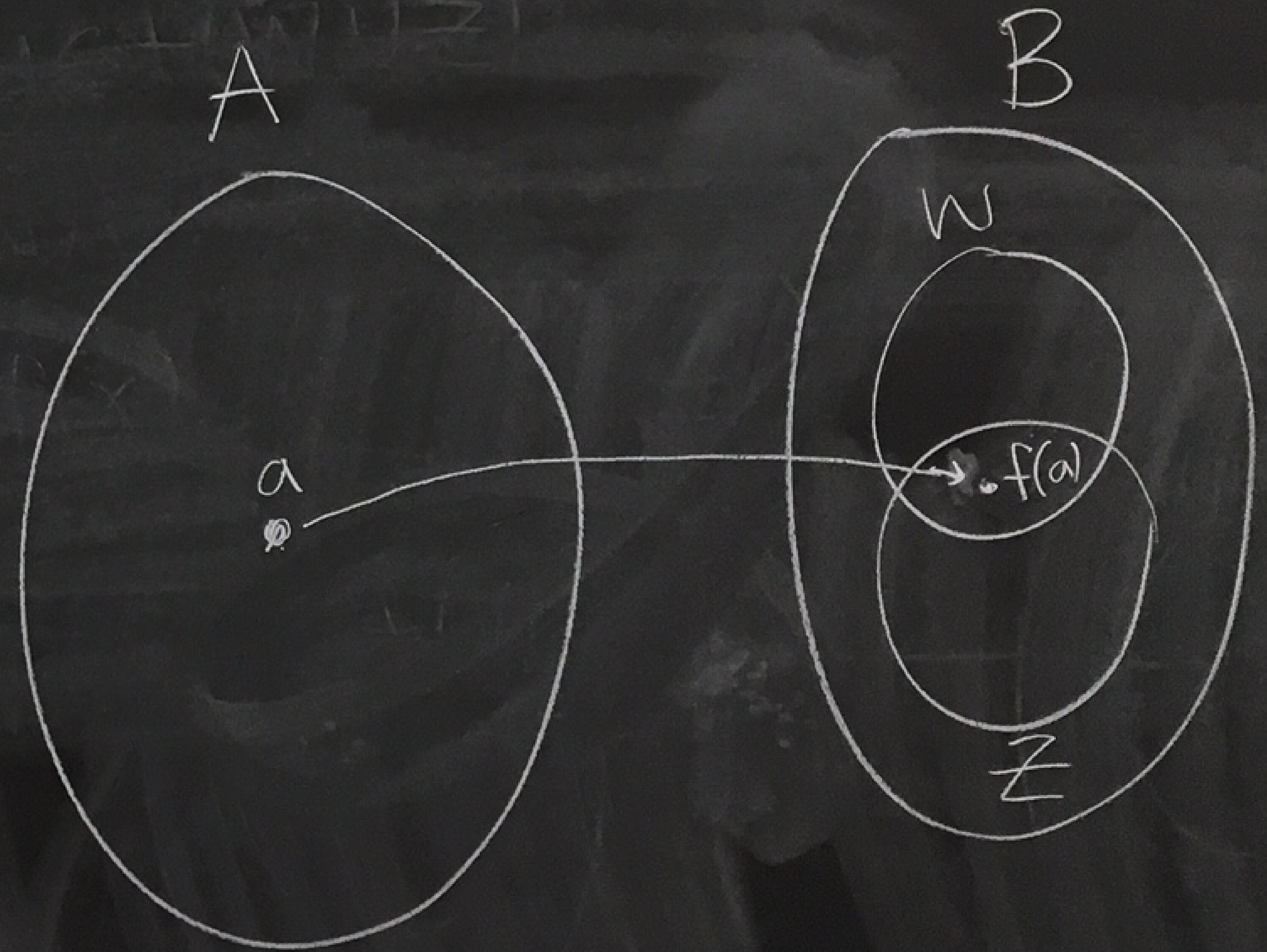
Then $f(a) \in W \cap Z$.

So, $f(a) \in W$ and $f(a) \in Z$.

Since $f(a) \in W$ we have $a \in f^{-1}(W)$.

Since $f(a) \in Z$ we have $a \in f^{-1}(Z)$.

Thus, $a \in f^{-1}(W) \cap f^{-1}(Z)$.



$\boxed{\supseteq}$: Let $a \in f^{-1}(W) \cap f^{-1}(Z)$.

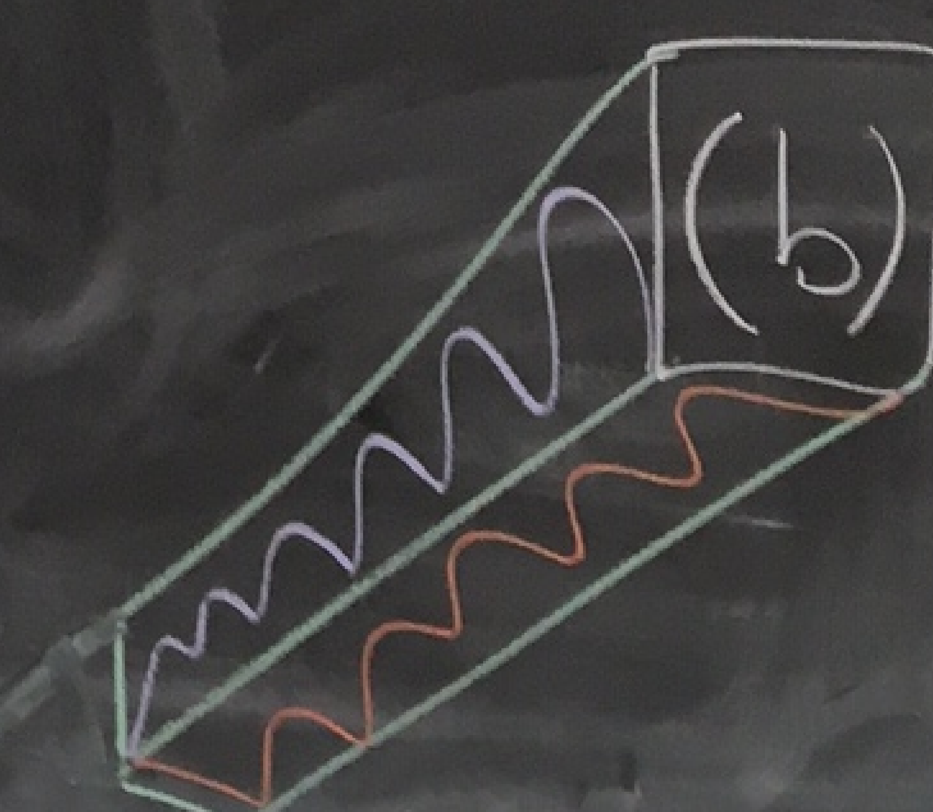
So, $a \in f^{-1}(W)$ and $a \in f^{-1}(Z)$.

Thus, $f(a) \in W$ and $f(a) \in Z$.

Hence, $f(a) \in W \cap Z$.

Therefore, $a \in f^{-1}(W \cap Z)$.

So, $f^{-1}(W) \cap f^{-1}(Z) \subseteq f^{-1}(W \cap Z)$.



(c) $Y \subseteq B$, $A - f^{-1}(Y) \subseteq f^{-1}(B-Y)$.

Let $a \in A - f^{-1}(Y)$.

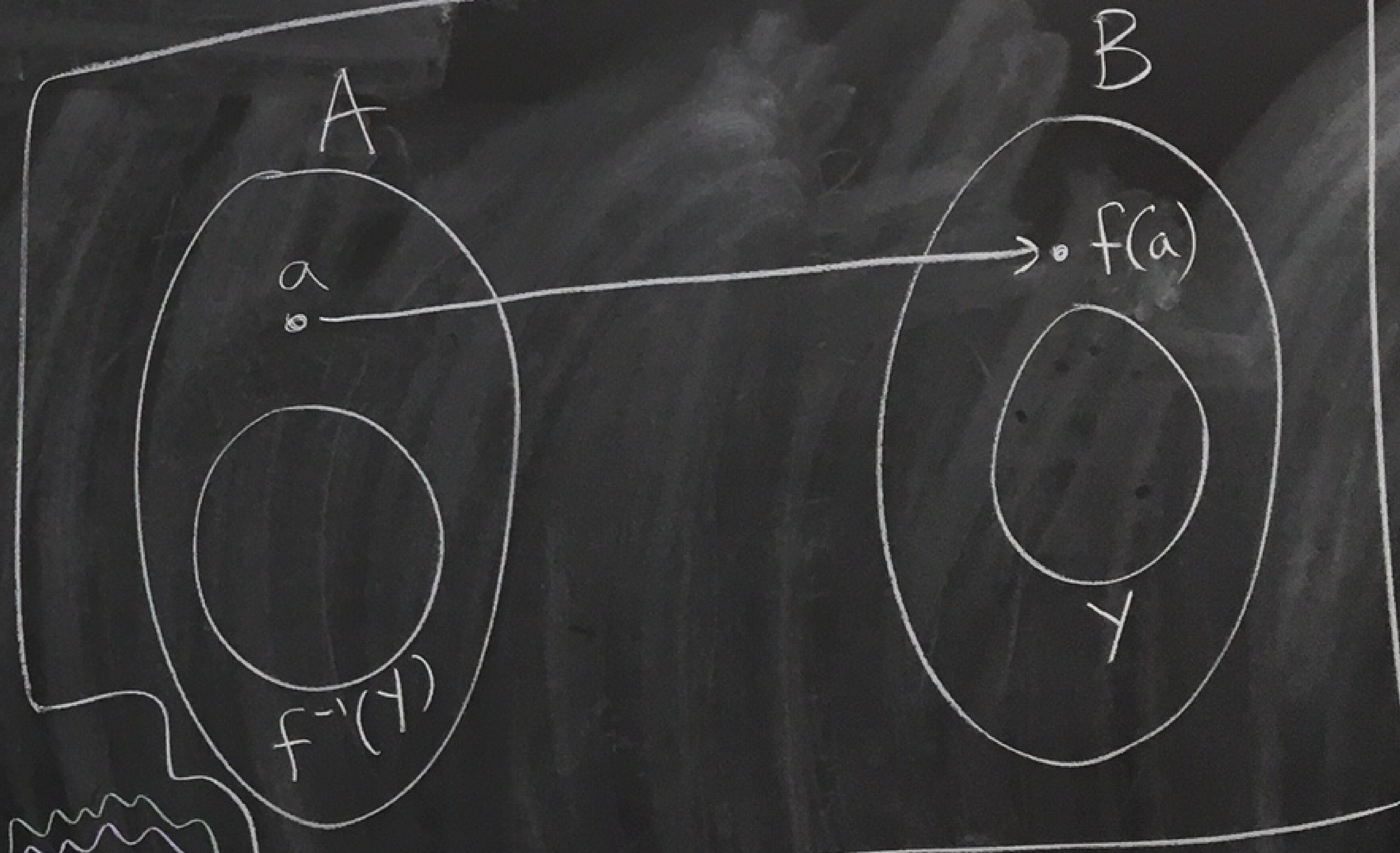
So, $a \in A$ and $a \notin f^{-1}(Y)$.

Thus, $f(a) \in B$ and $f(a) \notin Y$.

So, $f(a) \in B - Y$.

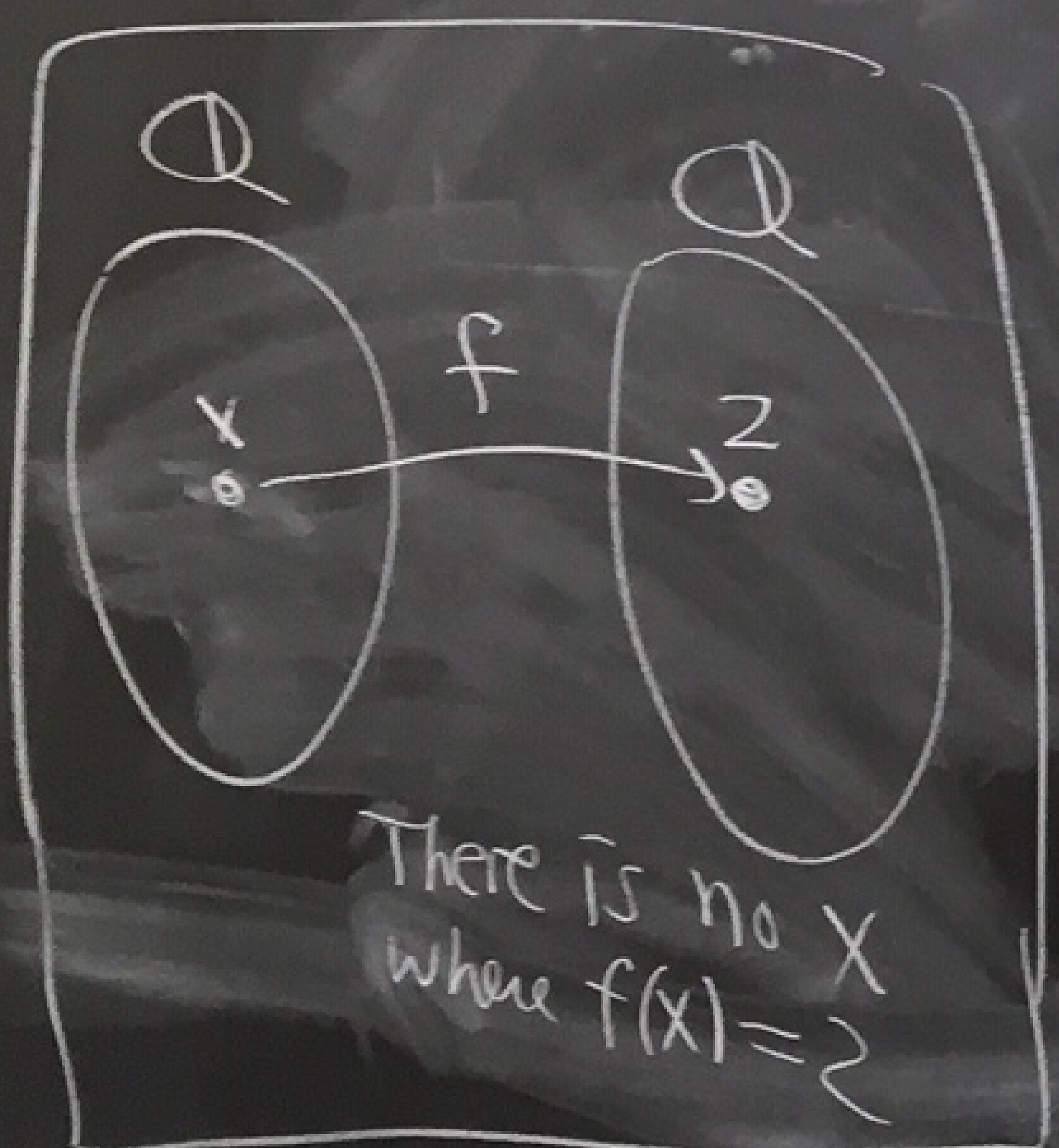
Thus, $a \in f^{-1}(B-Y)$.

Therefore, $A - f^{-1}(Y) \subseteq f^{-1}(B-Y)$. $\boxed{(c)}$



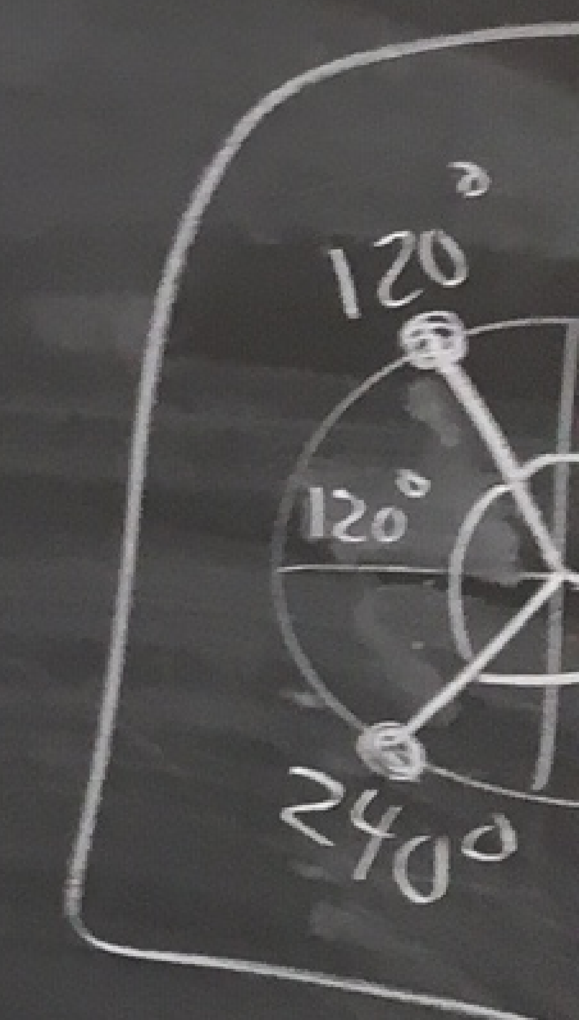
Hw 4

2(b) $\boxed{\begin{array}{l} f: \mathbb{Q} \to \mathbb{Q} \\ f(x) = x^3 \end{array}}$



(1-1 ?) Given $x, y \in \mathbb{Q}$, if $f(x) = f(y)$
then $x^3 = y^3$. So, $x = y$.
So, f is 1-1.

(onto ?) No, f is not onto $\mathbb{Q}$.
For example, there is no $x \in \mathbb{Q}$ with $f(x) = 2$.
If so, there would be $x \in \mathbb{Q}$ with $x^3 = 2$.



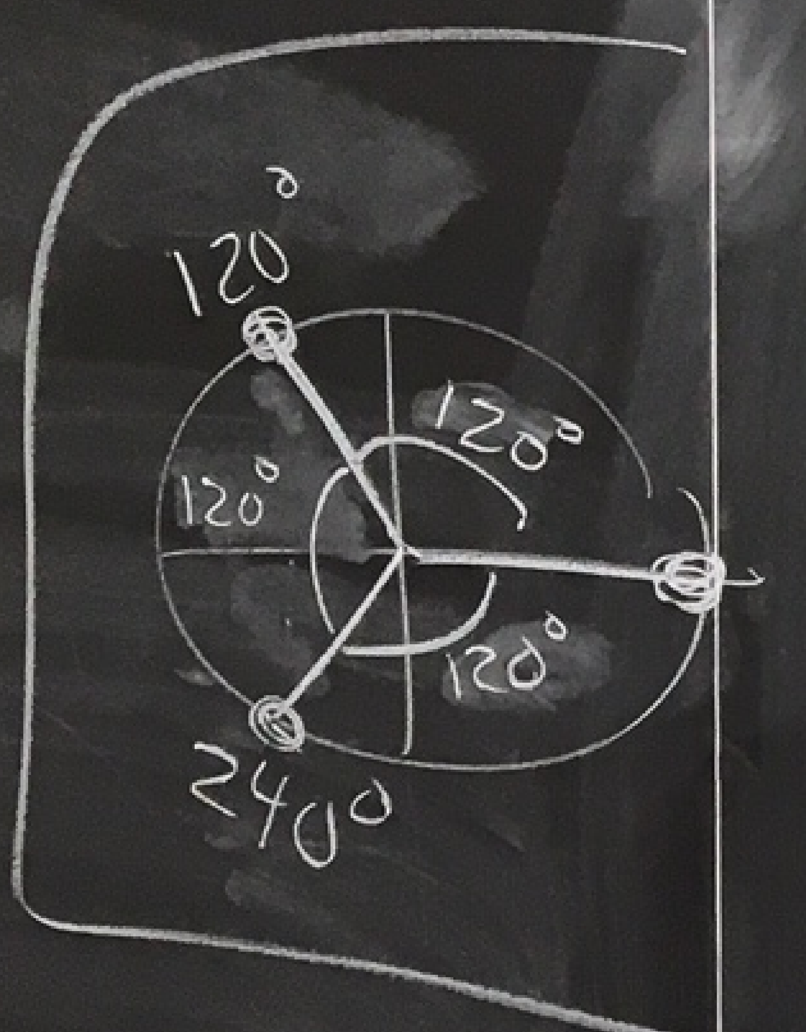
Either use HW technique to show there is no $x \in \mathbb{Q}$ with $x^3 = 2$.

Or say that the only solutions to $x^3 = 2$ are

$$x = 2^{1/3},\quad 2^{1/3}\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right),\quad 2^{1/3}\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$$

$$x = 2^{1/3},\quad 2^{1/3}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right),\quad 2^{1/3}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

None of these are in $\mathbb{Q}$.



$f(x) = 2$.

$h\ x^3 = 2$.

Test 2

- def statements
- computations
- proofs