11/13
Weds

Test 2 – next Weds – 11/20

M	W
	11/13 functions
11/18 Cardinality	11/20 Test 2
12/2 Cardinality	12/4 Cardinality
12/9 Review HW on cardinality	12/11 Final 12-2pm

HW 3 (Constructing $\mathbb{Q}$ out of $\mathbb{Z}$)

⑨ $S = \mathbb{Z} \times (\mathbb{Z} - \{0\})$

Define $\sim$ on S where $(a,b) \sim (c,d)$ iff $ad = bc$.

(a) Is $(1,5) \sim (-3,-15)$?

$(1)(-15) = (5)(-3)$

Yes.

Motivation

We are going to construct $\mathbb{Q}$.

$(1,2) \in S$ think of it as $\frac{1}{2}$

$(a,b) \in S$, $b \neq 0$, think of it as $\frac{a}{b}$

$\frac{a}{b} = \frac{c}{d}$ iff $ad = bc$

$$\begin{tikzcd} \mathbb{N} \arrow[d, bend left, "\text{problem 8}"] \\ \mathbb{Z} \arrow[d, bend left, "\text{problem 9}"] \\ \mathbb{Q} \end{tikzcd}$$

(b) Is $(1,1) \sim (3,5)$?

No because $(1)(5) \neq (1)(3)$.

(c) Show $\sim$ is an equivalence relation

<u>pf:</u> See solutions.

(d) List five elements from

$$\overline{(1,1)} = \{(1,1),(2,2),(5,5),(-2,-2),(-8,-8),\dots\} = \overline{(2,2)}$$

$$\overline{(2,3)} = \{(2,3),(4,6),(-2,-3),(-4,-6),(8,12),\dots\} = \overline{(8,12)}$$

(f) Define $\overline{(a,b)} \odot \overline{(c,d)} = \overline{(ac, bd)}$

Show $\odot$ is well-defined.

Step 1: Pick $(a,b),(c,d) \in S$.

So, $a,b,c,d \in \mathbb{Z}$ and $b \neq 0$ and $d \neq 0$.

$\mathbb{Z}$ is closed under multiplication so

$ac, bd \in \mathbb{Z}$.

Since $b \neq 0$ and $d \neq 0$, we know $bd \neq 0$.

So, $(ac, bd) \in S$.

Thus, $\overline{(a,b)} \odot \overline{(c,d)} = \overline{(ac, bd)} \in S/\sim$.

Recall $S/\sim$ means the set of equivalence classes

Motivation

(f) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

(e) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$S = \mathbb{Z} \times (\mathbb{Z} - \{0\})$

Step 2 Suppose $(a,b),(c,d),(e,f),(g,h) \in S$

and $\overline{(a,b)} = \overline{(e,f)}$ and $\overline{(c,d)} = \overline{(g,h)}$.

We need to show $\overline{(a,b)} \odot \overline{(c,d)} = \overline{(e,f)} \odot \overline{(g,h)}$

Since $\overline{(a,b)} = \overline{(e,f)}$ and $\overline{(c,d)} = \overline{(g,h)}$

we know $(a,b) \sim (e,f)$ and $(c,d) \sim (g,h)$.

Thus, $af = be$ and $ch = dg$.

Multiplying these equations gives $afch = bedg$.

So, $(ac)(fh) = (bd)(eg)$.

side-work

NTS:

$\overline{(ac,bd)} = \overline{(eg,fh)}$

$(ac,bd) \sim (eg,fh)$

$acfh = bdeg$

Th

Thus, $(ac, bd) \sim (eg, fh)$.

Hence, $\overline{(ac, bd)} = \overline{(eg, fh)}$.

Therefore,

$$\overline{(a,b)} \odot \overline{(c,d)} = \overline{(e,f)} \odot \overline{(g,h)}.$$

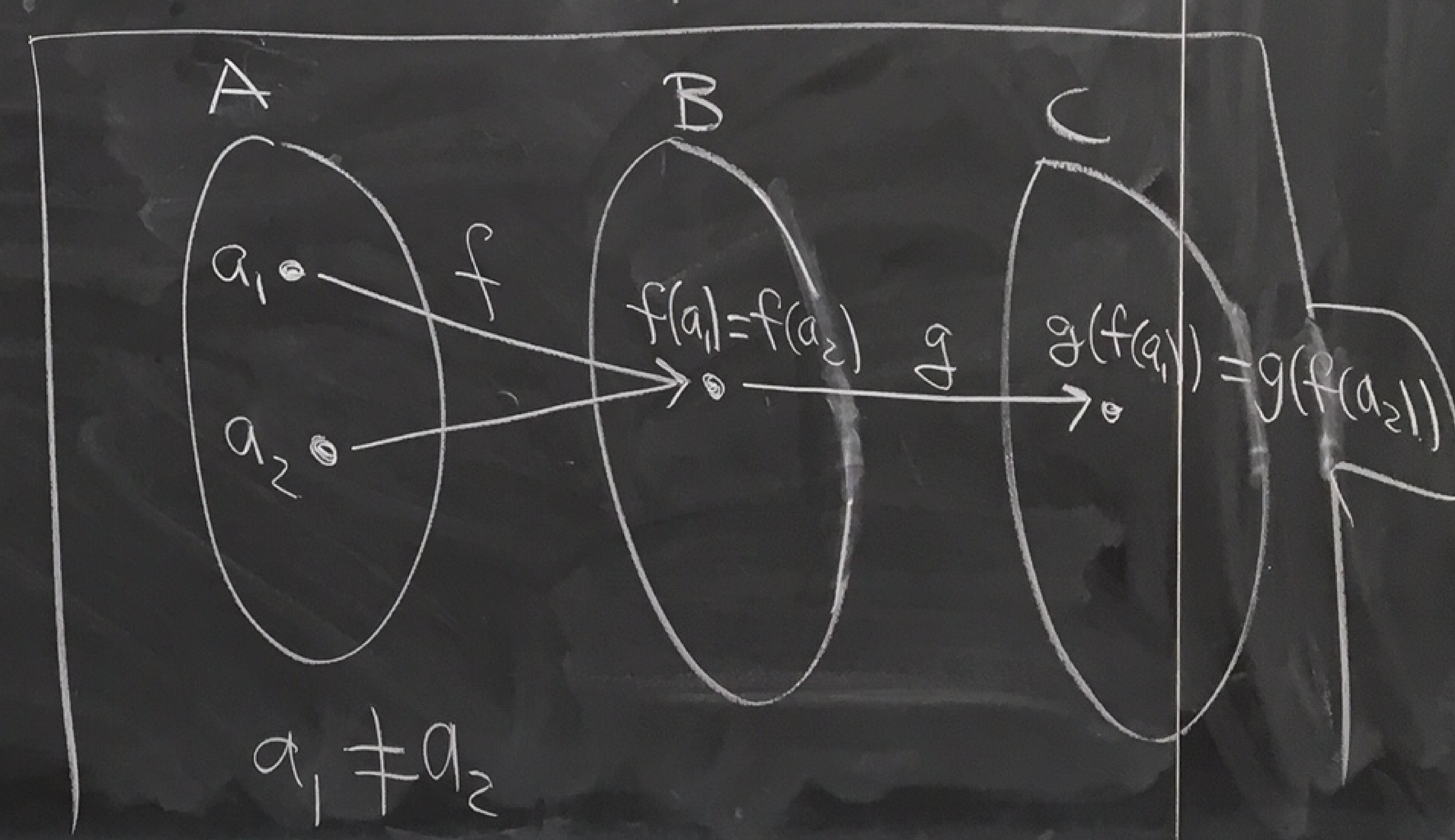
$\blacksquare$

HW 4

(9) Suppose $f: A \to B$ and $g: B \to C$.

Prove: If f is not one-to-one then $g \circ f$ is not one-to-one.

pf: Since f is not one-to-one there exist $a_1, a_2 \in A$ such that $f(a_1)=f(a_2)$ and $a_1 \neq a_2$.



Then, $g(f(a_1)) = g(f(a_2))$.

Hence $(g \circ f)(a_1) = (g \circ f)(a_2)$ and $a_1 \neq a_2$.

Therefore, $g \circ f$ is not one-to-one. $\blacksquare$

Then, $g(f(a_1)) = g(f(a_2))$.

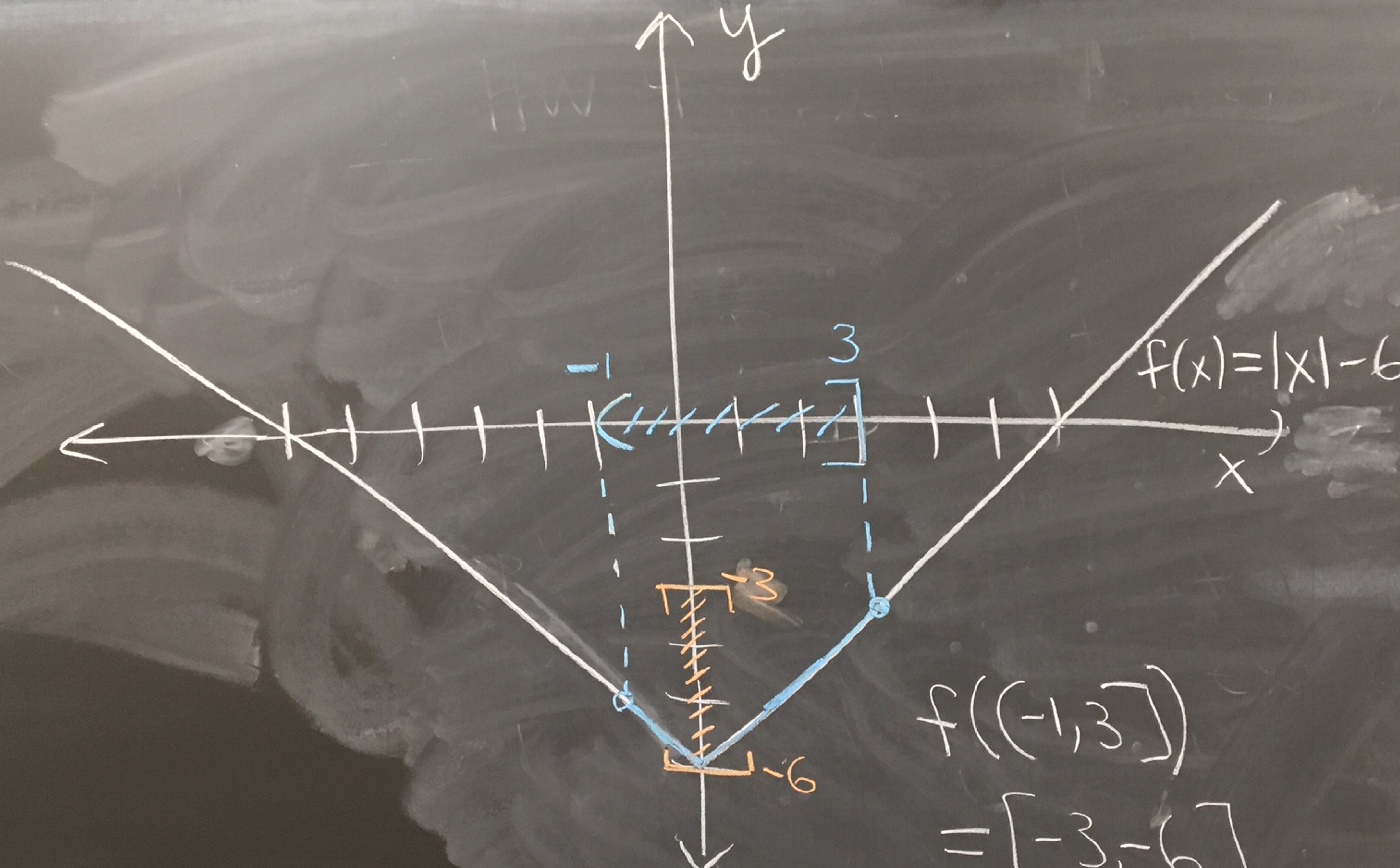
Hence $(g \circ f)(a_1) = (g \circ f)(a_2)$ and $a_1 \neq a_2$.

Therefore, $g \circ f$ is not one-to-one. $\blacksquare$

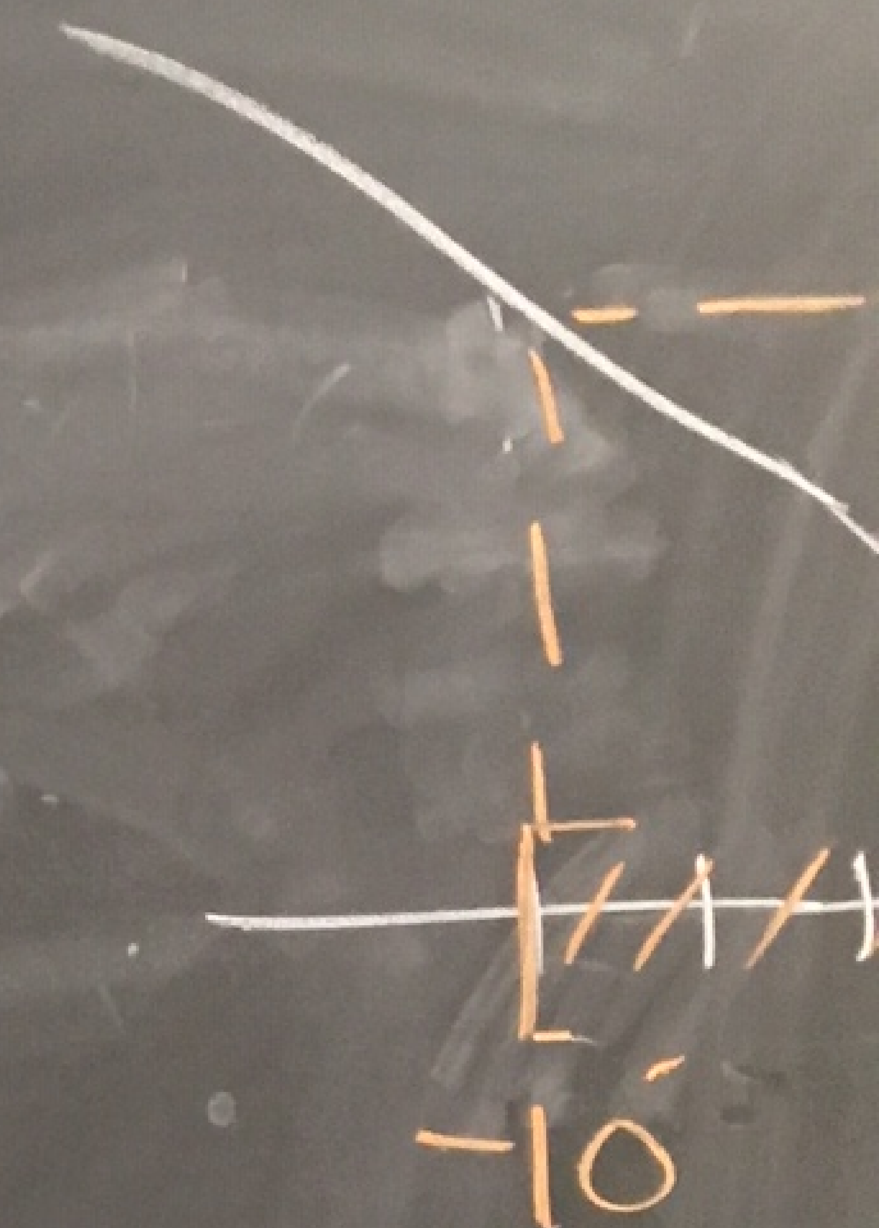
<u>Ex:</u> $f(x) = |x| - 6$

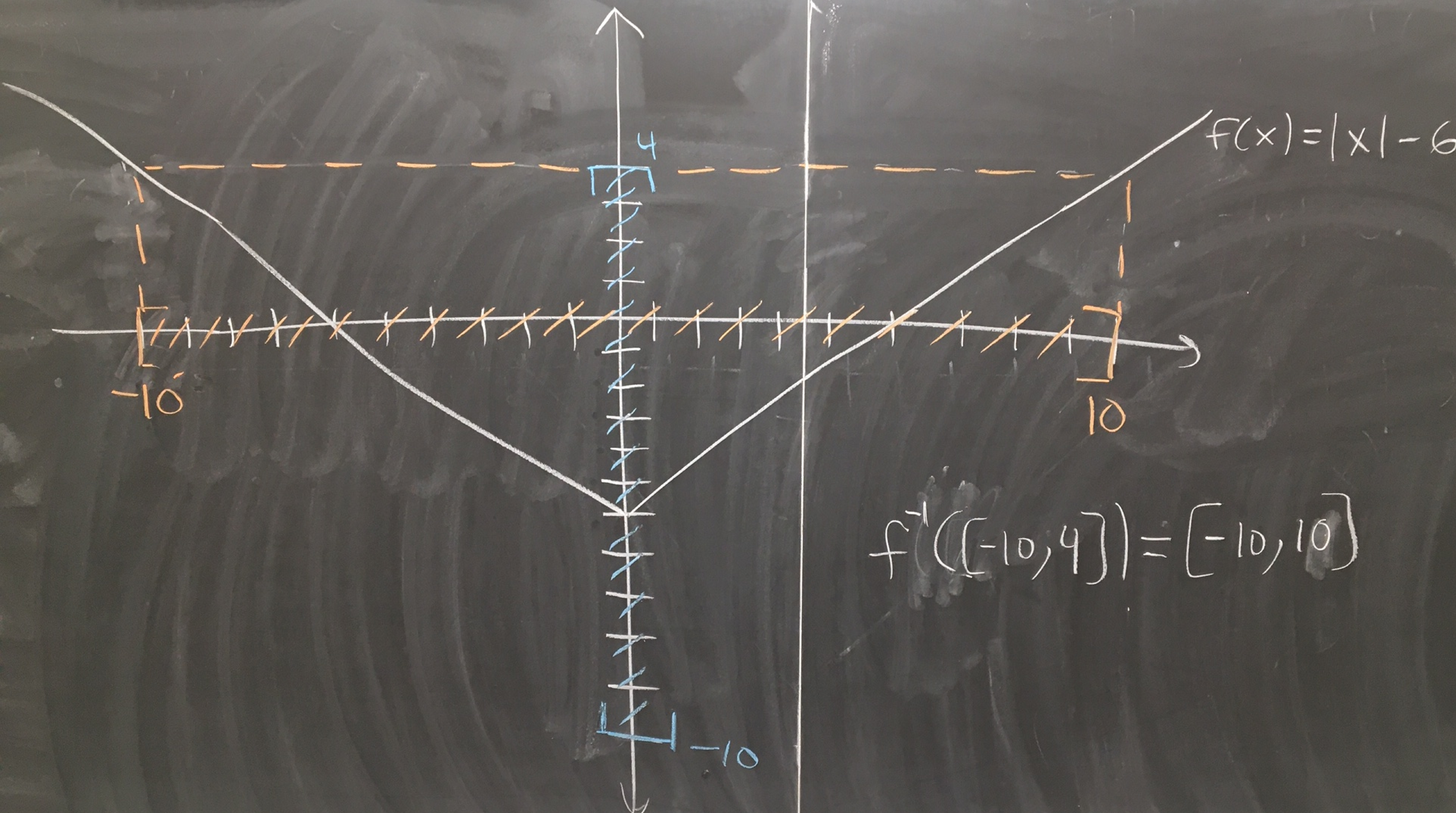
(a) $f((-1, 3])$

(b) $f^{-1}([-10, 4])$



$$f((-1,3]) = [-3,-6]$$





$$f^{-1}([-10,4]) = [-10,10]$$