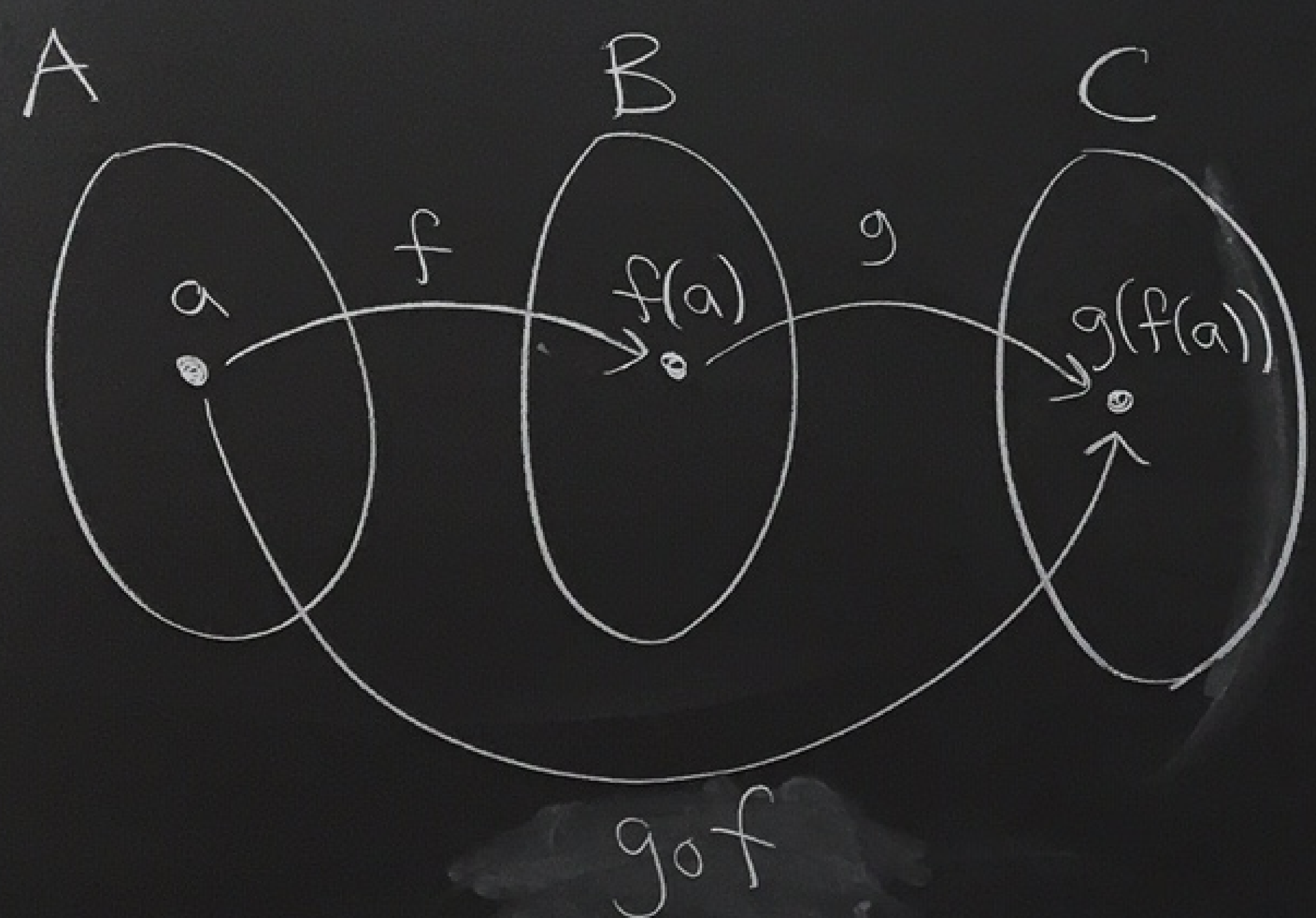
Weds 10/23

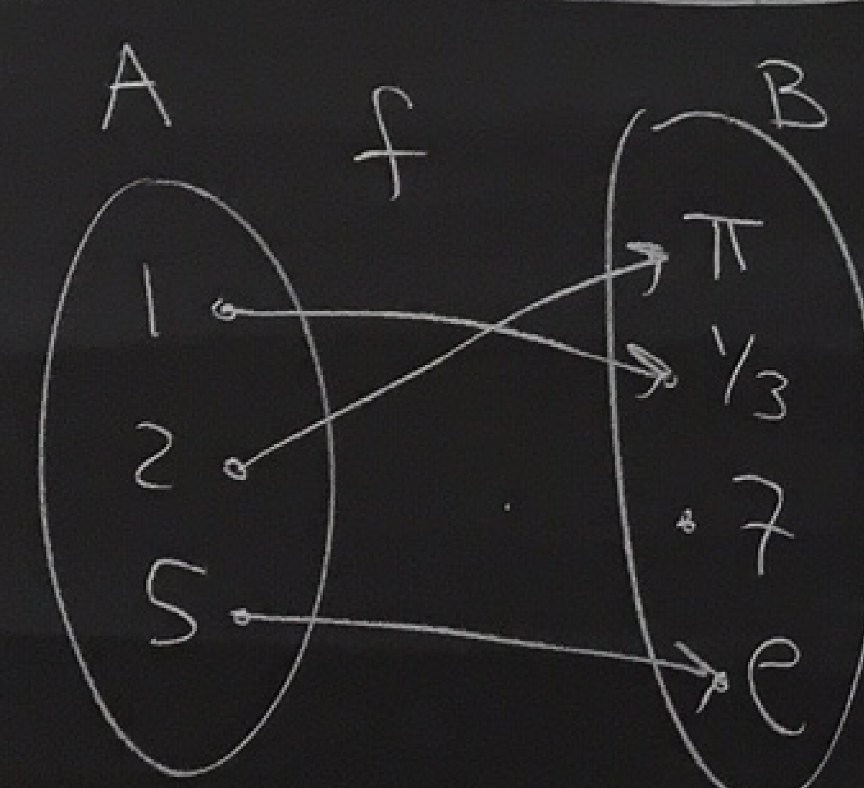


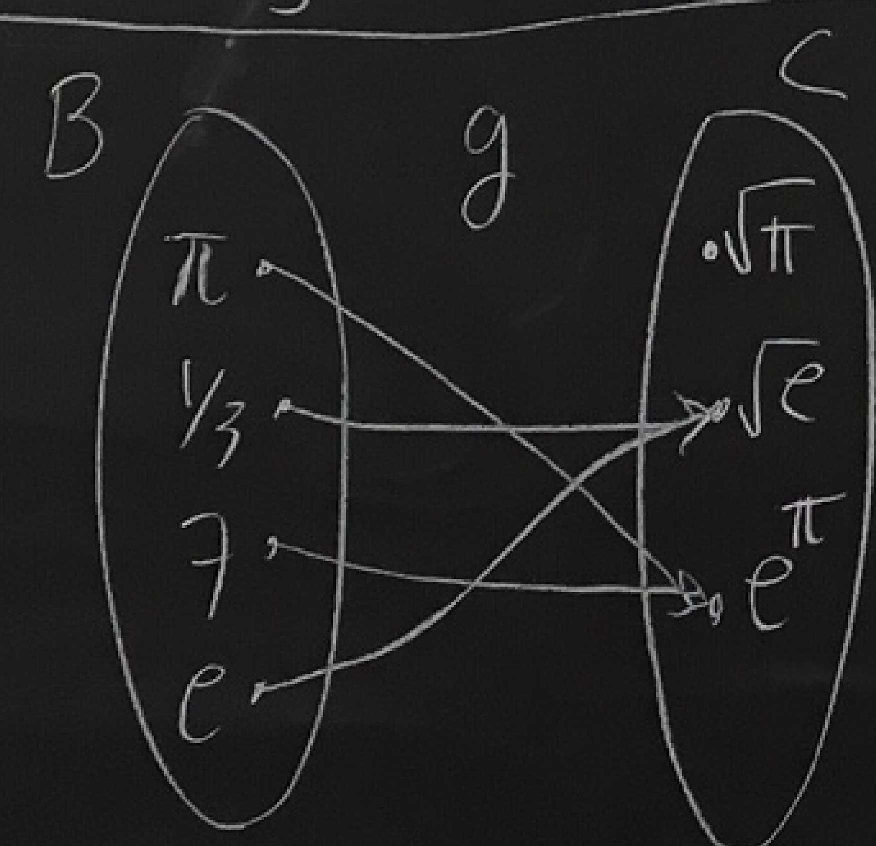
Def: Let A, B, and C be sets. Let $f: A \to B$ and $g: B \to C$. Define the <u>composition of f and g</u> to be the function $g \circ f : A \to C$ where $(g \circ f)(a) = g(f(a))$.

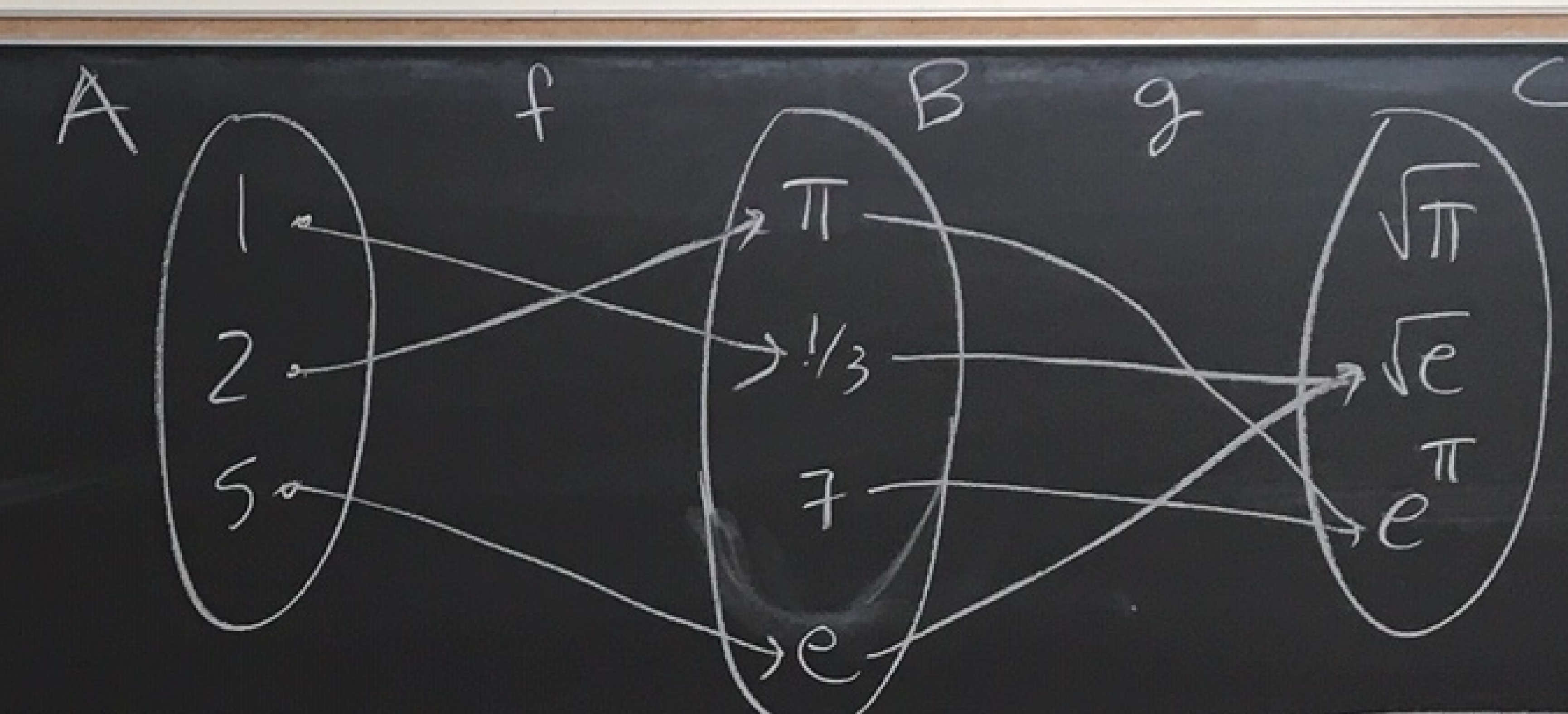
Ex: $A = \{1, 2, 5\}$

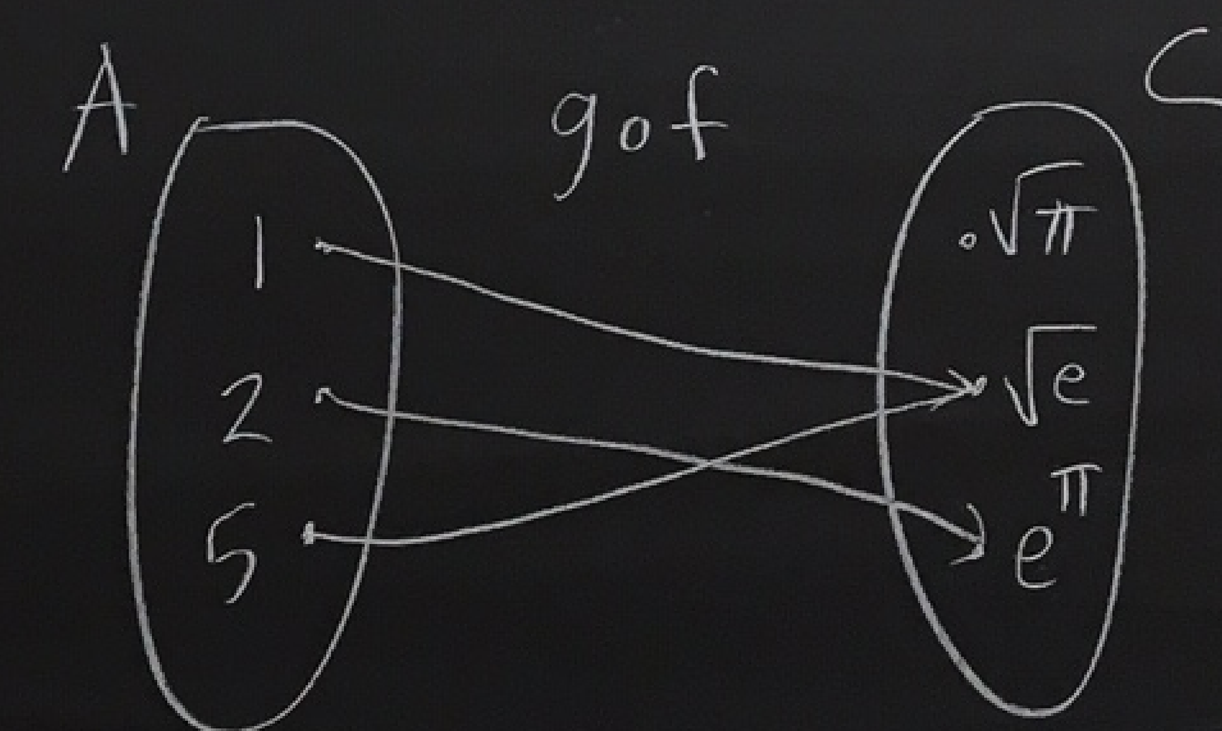
$B = \{\pi, \frac{1}{3}, 7, e\}$

$C = \{\sqrt{\pi}, \sqrt{e}, e^{\pi}\}$









$$(g \circ f)(1) = g(f(1)) = g(\tfrac{1}{3}) = \sqrt{e}$$

$$(g \circ f)(2) = g(f(2)) = g(\pi) = e^{\pi}$$

$$(g \circ f)(5) = g(f(5)) = g(e) = \sqrt{e}$$

Theorem: Let A, B, C be sets.

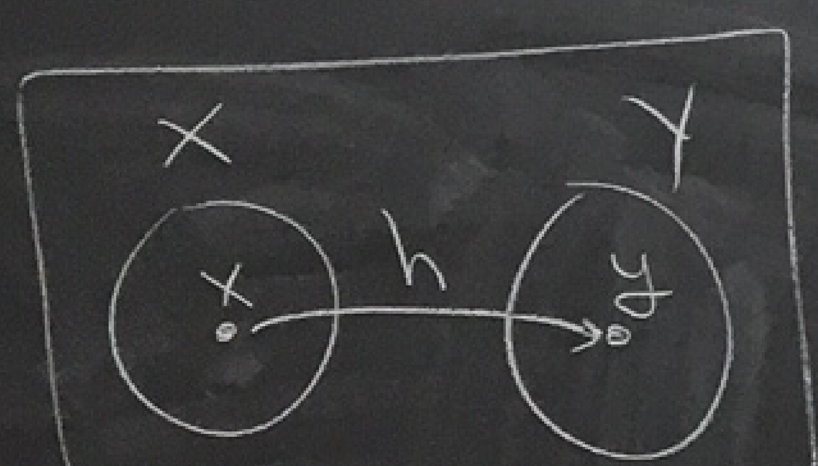
Let $f: A \to B$ and $g: B \to C$.

① If f and g are both onto, then $g \circ f$ is onto.

② If f and g are both one-to-one, then $g \circ f$ is one-to-one.

③ If f and g are both bijections then $g \circ f$ is a bijection.

bijection means one-to-one and onto



$h : X \to Y$.

prove h is onto

pf: Let $y \in Y$.

⋮

Find $x \in X$ with $h(x) = y$

pf:

① Assume that f and g are onto.
We want to show that $g \circ f$ is onto.

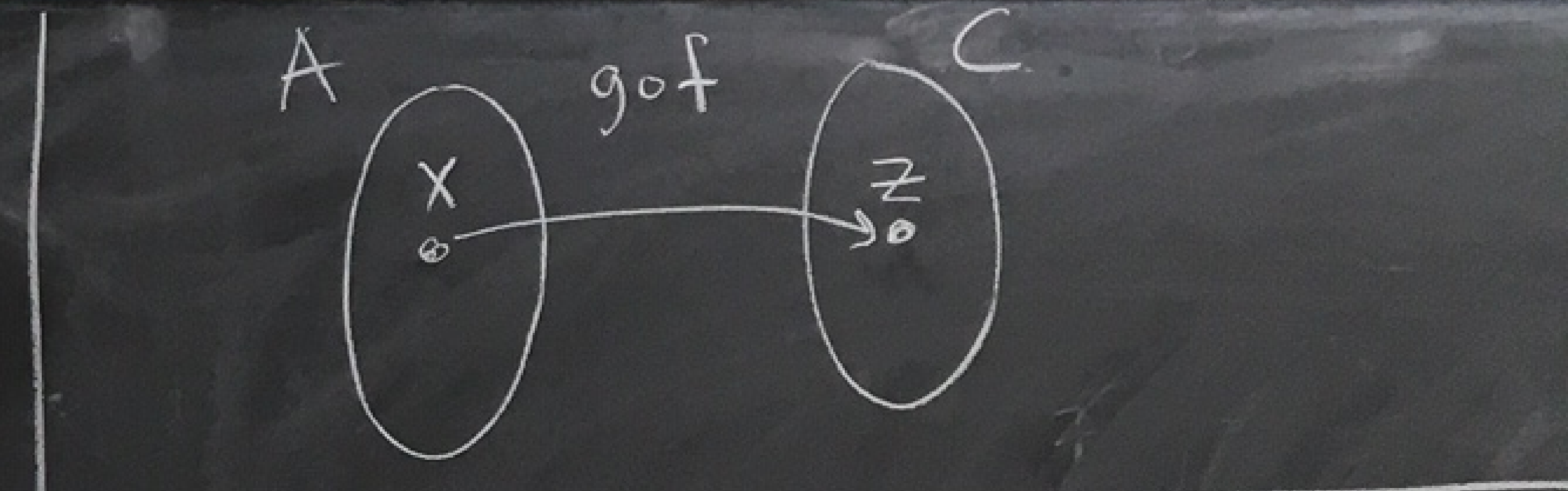
Let $z \in C$.

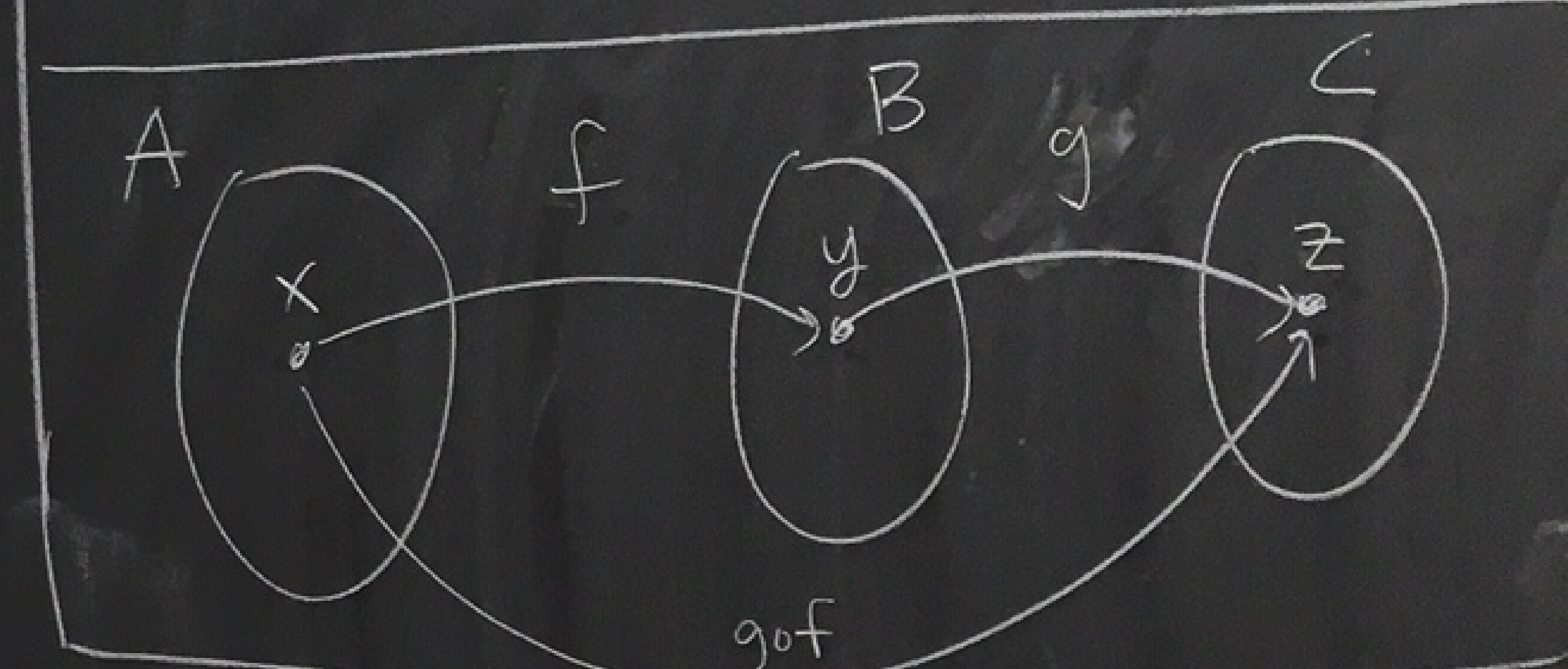
Since $g : B \to C$ is onto,
there exists $y \in B$
with $g(y) = z$.

Since $f : A \to B$ is onto,
there exists $x \in A$
with $f(x) = y$.

Then, $x \in A$ and $(g \circ f)(x) = g(f(x))$
$= g(y) = z$.

So, $g \circ f$ is onto.





② Assume f and g are both one-to-one

We want to prove that $g \circ f$ is one-to-one.

Suppose $a_1, a_2 \in A$ with $(g \circ f)(a_1) = (g \circ f)(a_2)$.

So, $g(f(a_1)) = g(f(a_2))$.

Since g is one-to-one and $g(\boxed{f(a_1)}) = g(\boxed{f(a_2)})$

we know $f(a_1) = f(a_2)$.

Since f is one-to-one and $f(\boxed{a_1}) = f(\boxed{a_2})$

we know $a_1 = a_2$.

Therefore, $g \circ f$ is one-to-one.

$h : X \to Y$
How to prove h is one-to-one

pf: Let $x_1, x_2 \in X$

Assume $h(x_1) = h(x_2)$.

⋮

Show $x_1 = x_2$.

pf:

① Assu

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Let

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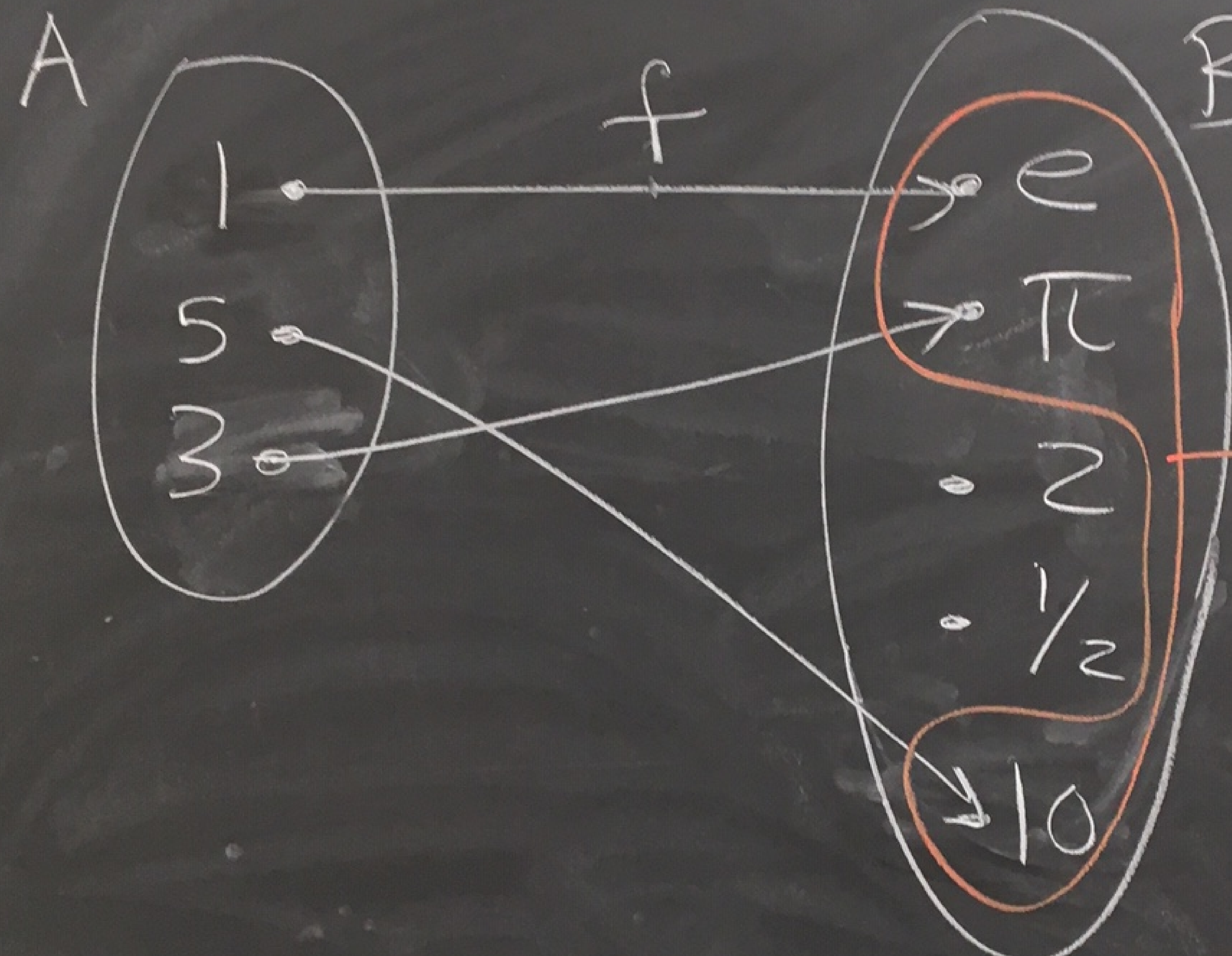
(3) Suppose f and g are both bijections (1-1 and onto).
By part 1, since f and g are both onto, we have $g \circ f$ is onto.
By part 2, since f and g are both 1-1, we have $g \circ f$ is 1-1.
So, $g \circ f$ is a bijection. $\blacksquare$

Ex: Consider this function f.



$C = \text{range}(f)$

$= \{e, \pi, 10\}$

f is one-to-one.

So we can make $f^{-1}: C \to A$

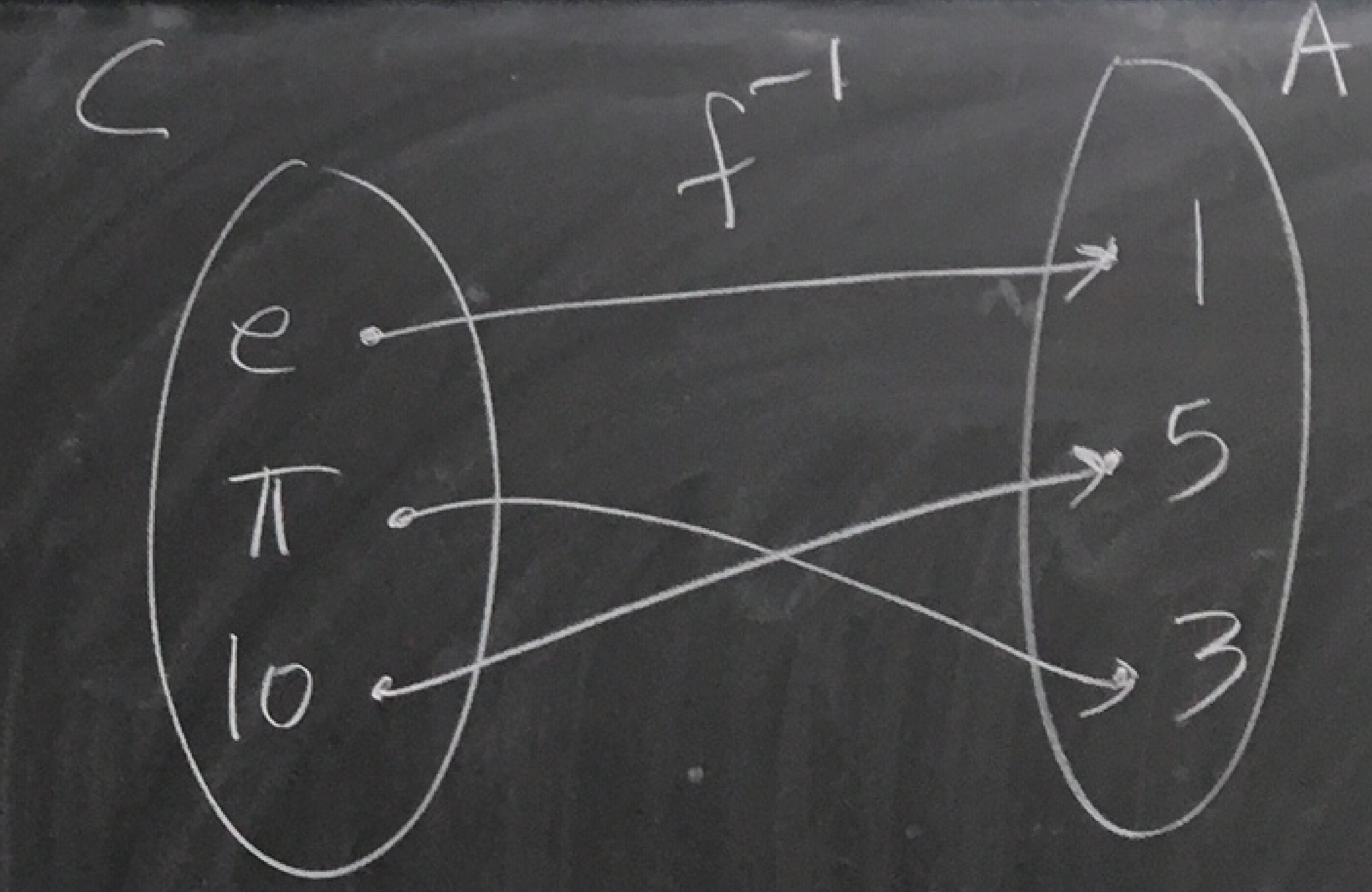
by reversing the arrows.

f^{-1} will be a well-defined

function because f is one-to-one

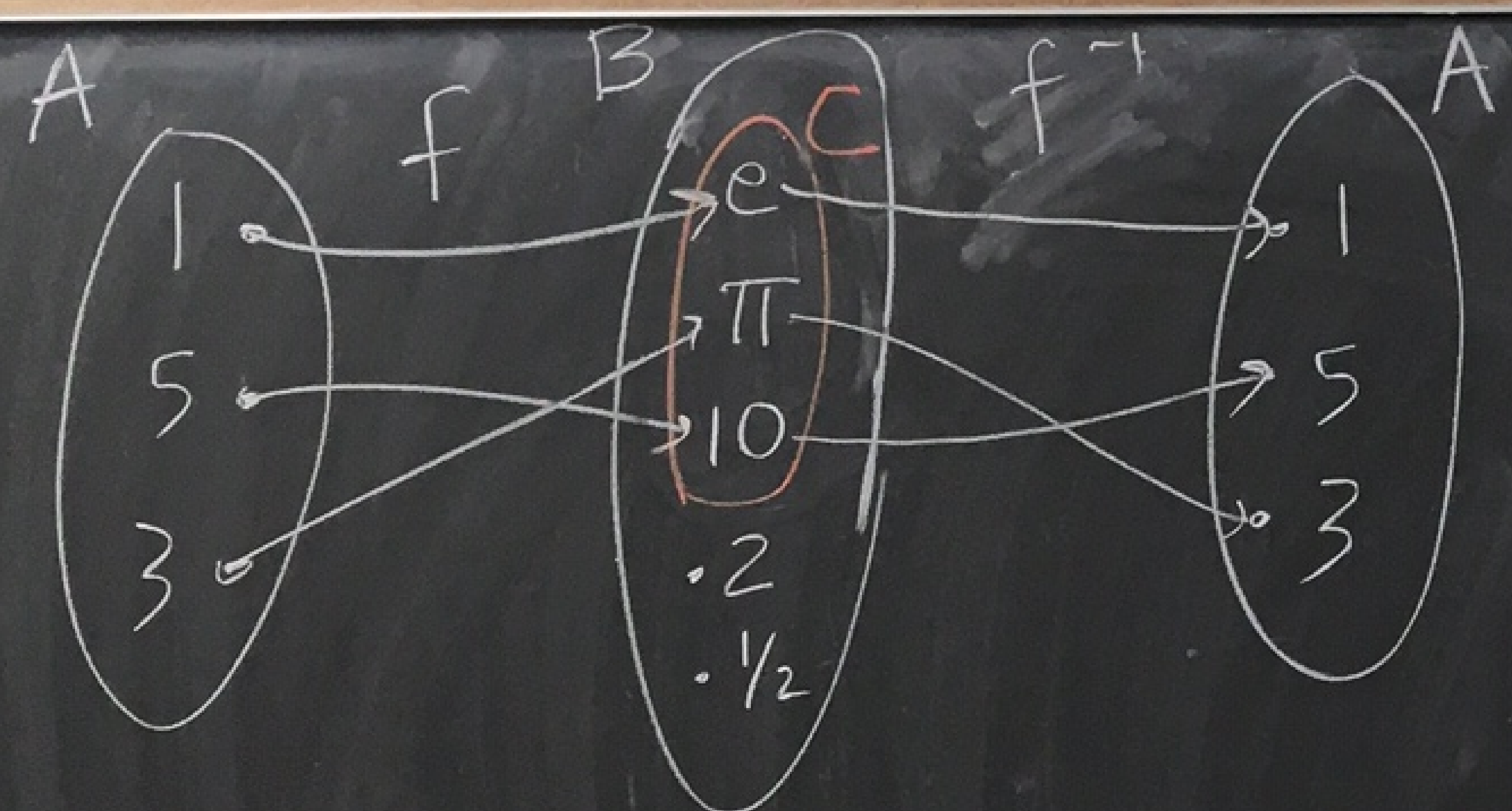
So there is only one arrow to

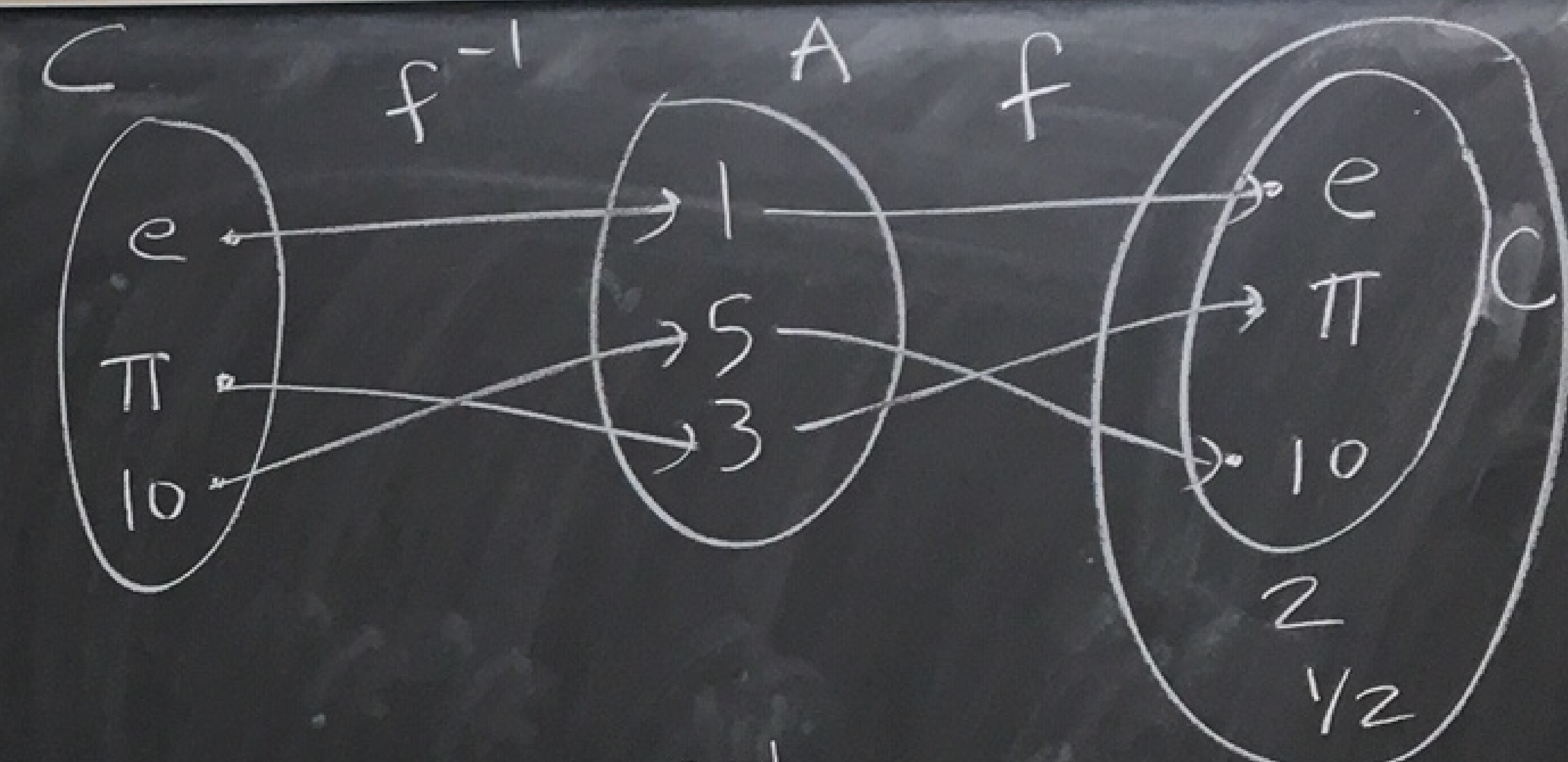
reverse at each element of C.

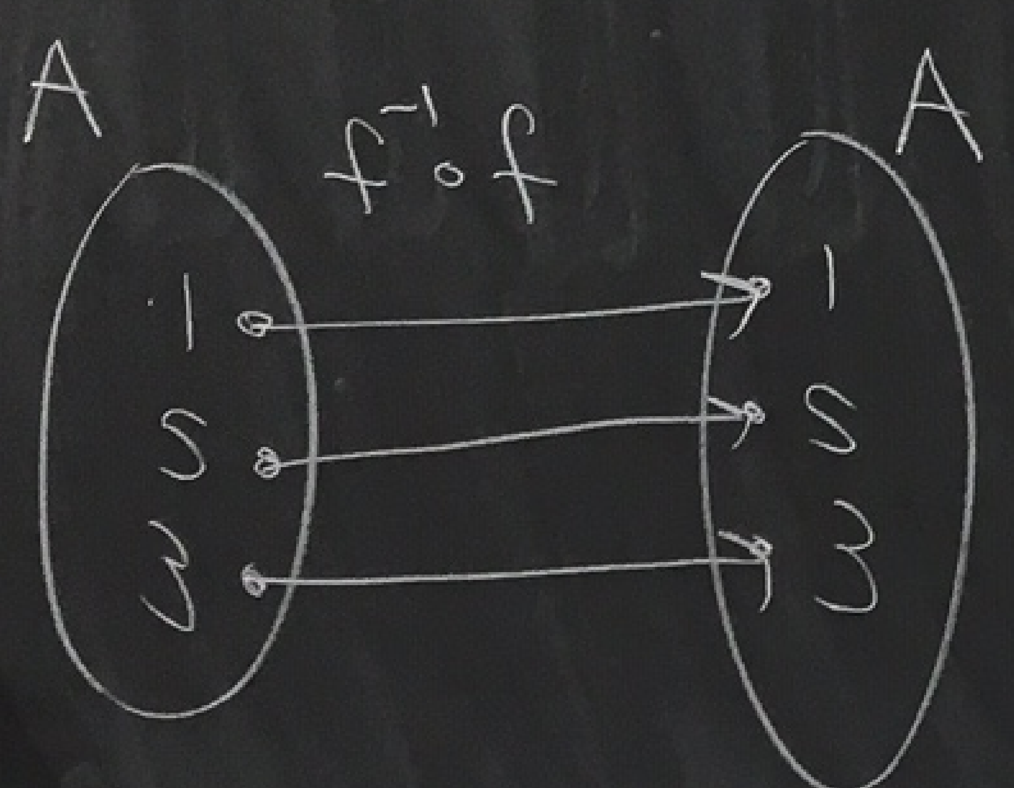


$$\text{domain}(f^{-1}) = C = \text{range}(f)$$

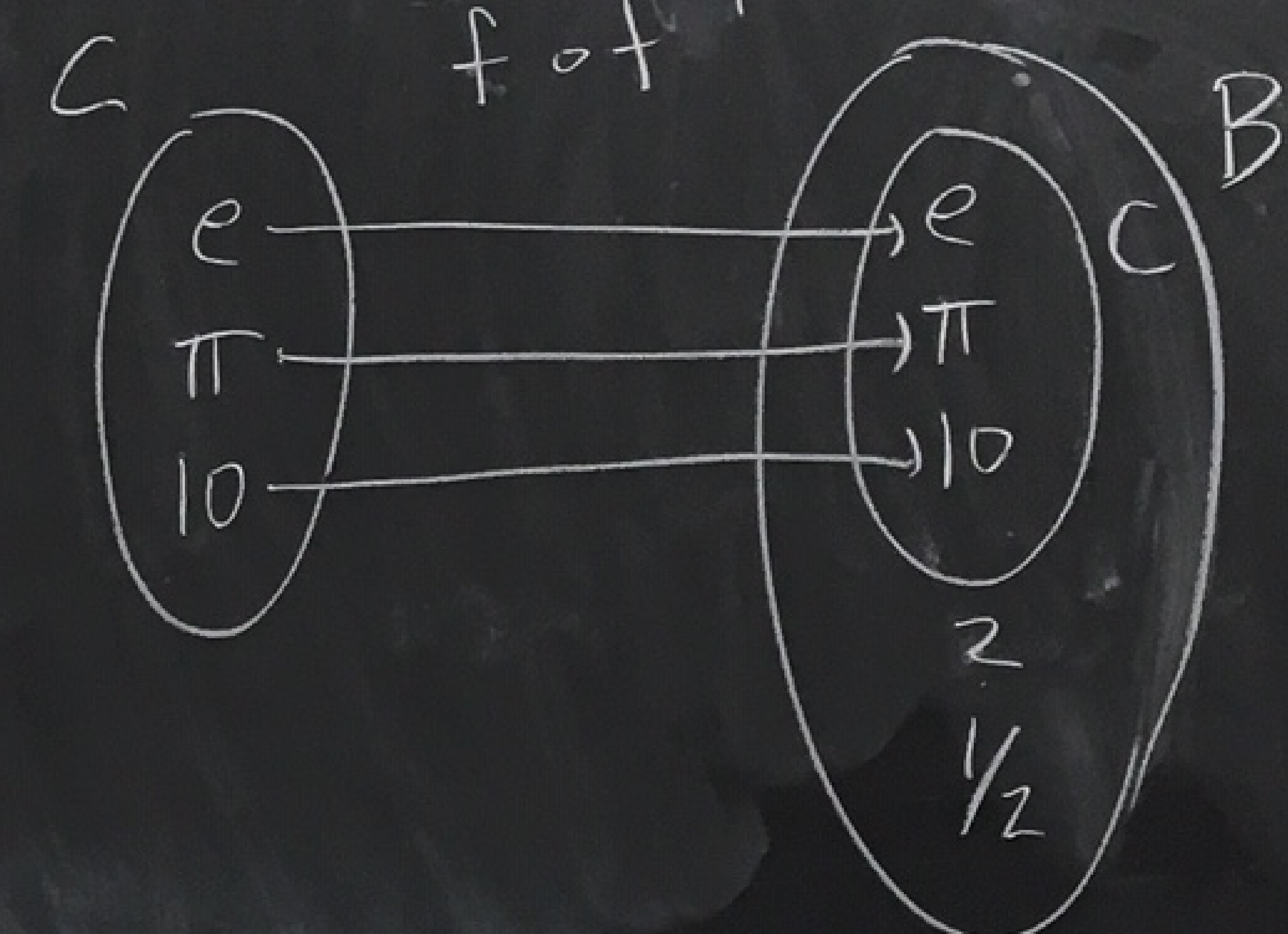
$$\text{range}(f^{-1}) = A = \text{domain}(f)$$







$f^{-1} \circ f = i_A$



$(f \circ f^{-1})(z) = z$
for all $z \in C$
$(f \circ f^{-1})(z) = i(z)$
for all $z \in C$.

Def: Let A and B be sets.

Let $f: A \to B$ be a one-to-one function. Let $C = \text{range}(f)$.

Define the *inverse function of f* to be $f^{-1}: C \to A$ where $f^{-1}(c) = a$ iff $f(a) = c$.

range$(f) = \{e, \pi, 10\}$

f^{-1} is well-defined since f is one-to-one

(that is, there is a unique a with $f(a) = c$ for all $c \in C = \text{range}(f)$)

M	W
f^{-1}	$f(X)$ $f^{-1}(Y)$
$\boxed{\text{\#W}}$	$\boxed{\text{review}}$
~~M~~	$\boxed{+2}$

M	W
cardinality	
$\boxed{\text{review}}$	$\boxed{F}$