Math 2150 - Test 1 - Spring 2025

Name:____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.

Score			
1		2	
3		4	
5		6	
Total			

Separable	$f(x) = g(y)\dfrac{dy}{dx}$	Separate $\dfrac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x,y) + N(x,y)\cdot y' = 0$	Test: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ Find f where $\dfrac{\partial f}{\partial x} = M$ and $\dfrac{\partial f}{\partial y} = N$ Solution: $f(x,y) = c$
Constant coefficient	$a_2y'' + a_1y' + a_0y = 0$	Two real roots r_1, r_2 use e^{r_1x} and e^{r_2x} If double real root r use e^{rx} and xe^{rx} If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x}\cos(\beta x)$ and $e^{\alpha x}\sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$ y_1 and y_2 sols. to homogeneous eqn.	$y_p = v_1y_1 + v_2y_2$ $v_1 = \int \dfrac{-y_2\cdot b(x)}{W(y_1,y_2)}dx \qquad v_2 = \int \dfrac{y_1\cdot b(x)}{W(y_1,y_2)}dx$
Reduction of Order	$y'' + a_1(x)y' + a_0(x)y = 0$ on the interval I	y_1 is a solution that isn't zero on I $y_2 = y_1 \cdot \int \dfrac{e^{-\int a_1(x)dx}}{y_1^2}dx$
Euler's method	$y' = f(x,y)$ $y(x_0) = y_0$	$x_n = x_{n-1} + h$ $y_n = y_{n-1} + h\cdot f(x_{n-1}, y_{n-1})$

$b(x)$	y_p guess for undetermined coefficients
constant	A
$5x-3$	$Ax+B$
$10x^2-x+1$	Ax^2+Bx+C
$\sin(6x)$	$A\cos(6x)+B\sin(6x)$
$\cos(6x)$	$A\cos(6x)+B\sin(6x)$
e^{3x}	Ae^{3x}
$(2x+1)e^{3x}$	$(Ax+B)e^{3x}$
x^2e^{3x}	$(Ax^2+Bx+C)e^{3x}$
$e^{3x}\sin(4x)$	$Ae^{3x}\cos(4x)+Be^{3x}\sin(4x)$
$e^{3x}\cos(4x)$	$Ae^{3x}\cos(4x)+Be^{3x}\sin(4x)$

Taylor series: $f(x_0) + f'(x_0)(x-x_0) + \dfrac{f''(x_0)}{2!}(x-x_0)^2 + \dfrac{f^{(3)}(x_0)}{3!}(x-x_0)^3 + \dfrac{f^{(4)}(x_0)}{4!}(x-x_0)^4 + \dots$

$\int u\,dv = uv - \int v\,du \qquad \int \sin(x)dx = -\cos(x) \qquad \int \cos(x)dx = \sin(x) \qquad W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

1. **[5 points]** Consider the following ODE:

$$5x^2y''' - 3xy' + y = \sin(x)$$

(a) What is the order of the equation?

(b) Is it linear or not linear?

---

2. **[5 points]** Suppose you know that the general solution to $y'' - y' = 0$ is $y_h = c_1 + c_2e^x$ and that a particular solution to $y'' - y' = -3$ is $y_p = 3x$. What is the general solution to $y'' - y' = -3$?

3. [**10 points**] Solve the linear equation

$$y' + 3x^2 y = 3x^2$$

on $I = (-\infty, \infty)$

4. [10 points] Find a solution to the separable initial value problem

$$\frac{dy}{dx} = 6y^2x \qquad y(0) = 1$$

Solve for y in terms of x in your solution.

5. [10 points]

(a) Show that

$$(x^2 + y^2) + (2xy)y' = 0$$

is an exact equation.

(b) Find an implicit solution to the equation above in part (a).

6. [**10 points**] Find the general solution to

$$y'' - y' - 6y = 0$$