Math 4300 - Homework # 9

Interiors and the Crossbar Theorem

---

1. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Let $\ell \in \mathscr{L}$ be a line.

   (a) Let $S \subseteq \mathscr{P}$ be a line, ray, line segment, the interior of a ray, or the interior of a line segment. If $S \cap \ell = \emptyset$, then all the points of S lie on the same side of ℓ.

   (b) Let $A, B, C \in \mathscr{P}$ with $A - B - C$ and $\overleftrightarrow{AC} \cap \ell = \{B\}$. Then $\text{int}(\overrightarrow{BA})$ and $\text{int}(\overline{BA})$ both lie on the same side of ℓ, while $\text{int}(\overrightarrow{BA})$ and $\text{int}(\overrightarrow{BC})$ lie on opposite sides of ℓ.

---

2. **(This problem is used in Topic 11)**

   Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Suppose that $\angle AVB$ is an angle and B and P are on the same side of $\overleftrightarrow{VA}$. Prove that $P \in \text{int}(\angle AVB)$ if and only if A and B are on opposite sides of $\overleftrightarrow{VP}$

---

3. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Let $A, B, C, P \in \mathscr{P}$ where A, B, C are noncollinear. Prove: $P \in \text{int}(\angle ABC)$ if and only if A and P are on the same side of $\overleftrightarrow{BC}$, and C and P are on the same side of $\overleftrightarrow{BA}$.

---

4. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Let $A, B, C, P \in \mathscr{P}$ where A, B, C are noncollinear. Prove that if $A - P - C$, then $P \in \text{int}(\angle ABC)$ and $\text{int}(\overline{AC}) \subseteq \text{int}(\angle ABC)$.

---

5. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Given an angle $\angle AVB$, show that if $\overrightarrow{VP} \cap \text{int}(\overline{AB}) \neq \emptyset$, then $P \in \text{int}(\angle AVB)$.

---

6. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Let $A, B, C, D, P \in \mathscr{P}$ where A, B, C are noncollinear. If $A - B - D$, then $P \in \text{int}(\angle ABC)$ if and only if $C \in \text{int}(\angle DBP)$.

---

7. Let $(\mathscr{P}, \mathscr{L}, d)$ be a Pasch geometry. Let A, B, C be noncollinear points from $\mathscr{P}$. Prove that $\text{int}(\triangle ABC)$ is convex.