Math 474 - Homework # 6
More on Random Variables, Distributions, and Variance

1. Consider the experiment where you flip a coin 3 times. Let X denote the number of tails that occur.

   (a) Draw a picture of X and of of the probability function p of X.

   (b) Calculate $E[X]$ and $\mathrm{Var}[X]$.

2. Consider the experiment where you roll two 4-sided dice. Let X be the sum of the two dice.

   (a) Draw a picture of X and of the probability function p of X.

   (b) Draw a picture of the cumulative distribution function F of X.

   (c) Calculate $E[X]$ and $\mathrm{Var}[X]$ and $\sigma = \sigma_X$.

3. Consider the experiment where you roll two 4-sided dice. Let X be the maximum of the two dice.

   (a) Draw a picture of X and of the probability function p of X.

   (b) Draw a picture of the cumulative distribution function F of X.

   (c) Calculate $E[X]$ and $\mathrm{Var}[X]$ and $\sigma = \sigma_X$.

4. You are interested in two games: game A and game B.

   - In game A, you pick a number between 1 and 100. A ball is drawn randomly from a box with balls that are numbered between 1 and 100. If the ball with your number is drawn then you win \$74. Otherwise you loose \$1.
   - In game B, there are four numbers to choose from. They are 1, 2, 3, and 4. You pick a number. Then a ball is drawn from a bag containing balls numbered 1, 2, 3, and 4. If your number is selected, then you win \$2. Otherwise you loose \$1.

   Answer the following questions.

(a) For each game let X be the amount of money won or lost. Graph the probability function for X.

(b) What is the expected value and variance of game A ?

(c) What is the expected value and variance of game B ?

(d) What game should you play?

5. Let X be a discrete random variable. Let $\mu = E[X]$ and $\sigma^2 = \mathrm{Var}[X]$.

   (a) Let k be a positive real number. Use Chebyshev's inequality to show that $P(|X - \mu| \geq k\sigma) \leq \dfrac{1}{k^2}$

   (b) Show that $P(|X - \mu| \geq 2\sigma) \leq \dfrac{1}{4}$. [Note: This says that the probability that a data point is at least 2 standard deviations away from the mean (on either side) is at most 25%.

6. The binomial distribution applies when we are interested in the number of successes in a fixed number of Bernoulli trials. What if instead we studied how long it takes to get the first success in a series of Bernoulli trials. That is we look at the probability of having a string of failures (that is, multiple failures in a row) and then a success.

   More specifically, let $0 < p < 1$ and $q = 1 - p$. Consider the experiment where we do consecutive independent Bernoulli trials with probability p of success and q of failure. We repeat the experiment until we get the first success and then we stop.

   (a) What is a sample space S for this experiment? Let X be the number of trials until the first success occurs. Find a formula for $P(X = k)$. Note: X is called a Geometric random variable.

   (b) Sketch the probability function $p(k) = P(X = k)$ when the probability of success is $\frac{1}{2}$.

   (c) Show that $E[X] = \dfrac{1}{p}$ and $\mathrm{Var}[X] = \dfrac{1-p}{p^2} = \dfrac{q}{p^2}$.