Math 2550
HW 5 - Part 2
Determinants

1. Show that the following statements are not necessarily true by giving an example that shows that they can sometimes be false.

   In the statements, A and B are square matrices of the same size, and I is the identity matrix.

   (a) The deteminant of $I + A$ is $1 + A$.

   (b) $\det(AB - BA) = 0$

   (c) The determinant of $4\,A$ is $4 \cdot \det(A)$

   (d) $\det(A + B) = \det(A) + \det(B)$.

2. Show that the following statements are always true by proving it.

   (a) Let A be a 2×2 matrix and α be a real number. Prove that

   $$\det(\alpha \cdot A) = \alpha^2 \cdot \det(A)$$

   (b) Let A and B be 2×2 matrices. Prove that

   $$\det(AB) = \det(A)\det(B)$$

3. Let $A = \begin{pmatrix} a & b & c \\ 0 & 0 & d \\ 0 & 0 & e \end{pmatrix}$ where a, b, c, d, e are non-zero numbers.

   Show that $\det(A) = 0$.

4. Find a value of k such that the matrix $A = \begin{pmatrix} 1 & k & 1 \\ 2 & 1 & 2 \\ 1 & 0 & k \end{pmatrix}$ is not invertible.