Math 4300 - Homework # 5

Line segments and rays

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1. In the Euclidean plane, let $A = (-1, 2)$ and $B = (3, 8)$.
   (a) Draw an accurate picture of $\overline{AB}$.
   (b) Draw an accurate picture of $\overrightarrow{AB}$.
   (c) Draw an accurate picture of $\overrightarrow{BA}$.

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2. In the hyperbolic plane, let $A = (1, 2)$ and $B = (1, 4)$.
   (a) Draw an accurate picture of $\overline{AB}$.
   (b) Draw an accurate picture of $\overrightarrow{AB}$.
   (c) Draw an accurate picture of $\overrightarrow{BA}$.

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3. In the hyperbolic plane, let $A = (1, 2)$ and $B = (3, 4)$.
   (a) Draw an accurate picture of $\overline{AB}$.
   (b) Draw an accurate picture of $\overrightarrow{AB}$.
   (c) Draw an accurate picture of $\overrightarrow{BA}$.

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4. In the Euclican plane, let $P = (-2, 1)$, $Q = (-2, 3)$, $A = (0, 0)$, and $B = (2, 1)$.

   Find C on the ray $\overrightarrow{AB}$ such that $\overline{AC} \simeq \overline{PQ}$. Draw a picture of everything.

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5. In the hyperbolic plane, let $P = (1, 2)$, $Q = (1, 4)$, $A = (0, 2)$, and $B = (1, \sqrt{3})$.

   Find C on the ray $\overrightarrow{AB}$ such that $\overline{AC} \simeq \overline{PQ}$. Draw a picture of everything.

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6. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let A and B be distinct points from $\mathscr{P}$. Prove the following:

   (a) $\overline{AB} = \overline{BA}$

   (b) $\overline{AB} \subseteq \overrightarrow{AB} \subseteq \overleftrightarrow{AB}$

   (c) $\overline{AB} = \overrightarrow{AB} \cap \overrightarrow{BA}$

   (d) $\overleftrightarrow{AB} = \overrightarrow{AB} \cup \overrightarrow{BA}$

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7. (Segment Addition) Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.

   Let $A, B, C, P, Q, R \in \mathscr{P}$. Prove that if $A - B - C$, $P - Q - R$, $\overline{AB} \simeq \overline{PQ}$, and $\overline{BC} \simeq \overline{QR}$, then $\overline{AC} \simeq \overline{PR}$.

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8. (Segment Subtraction) Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.

   Let $A, B, C, P, Q, R \in \mathscr{P}$. Prove that if $A - B - C$, $P - Q - R$, $\overline{AB} \simeq \overline{PQ}$, and $\overline{AC} \simeq \overline{PR}$, then $\overline{BC} \simeq \overline{QR}$.

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9. Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let $A, B, C, D \in \mathscr{P}$ with $A \neq B$ and $C \neq D$. Prove that:

   (a) If $C \in \overrightarrow{AB}$ and $C \neq A$, then $\overrightarrow{AC} = \overrightarrow{AB}$.

   (b) If $\overrightarrow{AB} = \overrightarrow{CD}$, then $A = C$.

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10. In the Euclidean plane $\mathscr{E} = (\mathbb{R}^2, \mathscr{L}_E, d_E)$. Let $A, B \in \mathbb{R}^2$ with $A \neq B$.

    (a) Prove that

    $$\overline{AB} = \{C \in \mathbb{R}^2 \mid C = A + t(B - A) \text{ for some } t \text{ with } 0 \leq t \leq 1\}$$

(b) Prove that

$$\overrightarrow{AB} = \{C \in \mathbb{R}^2 \mid C = A + t(B - A) \text{ for some } t \text{ with } 0 \leq t\}$$

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11. Consider the hyperbolic plane $\mathscr{H} = (\mathbb{H}, \mathscr{L}_H, d_H)$. Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$ both be on the line ${}_cL_r$. Suppose that $x_1 < x_2$. Show that if $C = (x, y)$ lies on the line ${}_cL_r$ and $x_1 < x < x_2$, then $C \in \overline{AB}$.
