Math 474 - Homework # 4
Random Variables, Expected Value, Games

1. The following game is called Chuck-a-luck. It works as follows. You pick a number out of 1, 2, 3, 4, 5, or 6 and bet \$1 on that number. Three giant 6-sided dice are then rolled in a spinning cage. You then win \$1 for every time that your number appears on the dice. But you lose your \$1 if your number doesn't appear at all. For example, suppose that you you pick the number 1 as your number. Suppose that the dice show 1, 5, 1. Then you win \$2. If the dice showed 3, 1, 6 then you would win \$1. If the dice showed 3, 2, 6, then you would lose your \$1 bet.

   (a) Let X denote the amount of money lost or won. Let $p(i) = P(X = i)$ be the probability function for X. Calculate $p(-1)$, $p(1)$, $p(2)$, $p(3)$.

   (b) Draw a picture of p.

   (c) Draw a picture of the cumulative distribution function F where $F(i) = P(X \leq i)$.

   (d) What is the expected value of this game?

2. Consider the following experiment. Suppose we roll an 4-sided dice continually. We don't stop until a 3 is rolled.

   (a) What is a sample space S and a probability function P for such an experiment? Verify that you have a probability space.

   (b) Let A be the event that a 3 is rolled on the 3rd roll. Calculate $P(A)$.

   (c) Let B be the event that a 3 is rolled within the first 3 rolls. Calculate $P(B)$.

   (d) Suppose someone says this before the experiment starts: If a 3 is rolled within the first 3 rolls then I will pay you \$5, but if it doesn't then you have to pay me \$6. Do you take the bet?

3. Suppose there is a 4-sided die, but it isn't fair. Through experimentation you discover that a 1 is rolled approximately twice every eight rolls, a 2 is rolled approximately once every eight rolls, a 3 is rolled approximately three times every eight rolls, and a 4 is rolled approximately twice every eight rolls.

   (a) Suppose you win \$2 for every 1 or 2 that is rolled, but lose \$1 for every 3 or 4 that is rolled. What is the expected value of such a game?

   (b) What should you win or lose so that the game becomes "fair?" That is, so that the expected value is 0.

4. Suppose from a standard 52-card deck you are given the following five cards: 4♡, 10♡, Q♡, 3♠, 2♣. Suppose you now discard the 3♠ and 2♣ from your hand (but keep the other three cards) and ask for two more cards.

   (a) What are the odds you get two more hearts so you have a flush?

   (b) What are the odds you get one heart and a different suit?

   (c) What are the odds you get exactly one more queen?

   (d) What are the odds you get exactly two more queens?

   (e) What are the odds that you get at least one more queen?

   (f) Suppose someone says: If you get a flush I'll pay you \$500. But if you don't you have to pay me \$20. Do you take the bet?

5. Suppose that you flip a coin continually until a head occurs. Suppose that someone says: If you don't get a heads until you roll at least 3 tails then I'll pay you \$5. But if a heads occurs in the first three rolls then you must pay me \$1. Do you take the bet?

6. Two balls are chosen randomly at the same time from a bag containing 8 white balls, 4 black balls, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. You don't win or lose anything for each orange ball. Let X denote the winnings or loses of this game.

   (a) What are the possible values of X ? And what are the probabilities associated with each value?

(b) What is the expected value $E[X]$?

7. A gambling books recommends the following "winning strategy" for the game of roulette. Here we will use the American wheel that has 0 and 00 on it.

   It recommends that a gambler bet $1 on red. If red appears then the gambler should take their $1 profit and quit. If the gambler loses the bet, then they should play the game two more times and make additional bets of $1 on red on each of the next two spins of the roulette wheel and then quit. Let X denote the gambler's winnings or loses doing this strategy.

   (a) Find $P(X > 0)$, that is the probability that the gambler will win some money doing this.

   (b) Are you convinced that this is a "winning" strategy? Explain your answer. [Hint: Calculate $E[X]$]