Math 4300 - Homework # 3

More on the Euclidean plane

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1. In this problem we are working in the Euclidean plane $\mathscr{E} = (\mathbb{R}^2, \mathscr{L}_E)$. For the points A and B given, write down what L_{AB} is and draw a picture of L_{AB}. In your picture make sure to plot the values for $t = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, 2$. You may plot more if you wish to.

   (a) $A = (0, 0)$ and $B = (-1, -2)$

   (b) $A = (-2, 3)$ and $B = (1, 4)$

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2. Consider the Euclidean plane $\mathscr{E} = (\mathbb{R}^2, \mathscr{L}_E)$. Let $A, B, C \in \mathbb{R}^2$ and let $r, s \in \mathbb{R}$. Prove the following.

   (a) $A + B = B + A$

   (b) $(A + B) + C = A + (B + C)$

   (c) $r(A + B) = rA + rB$

   (d) $(r + s)A = rA + sA$

   (e) $\langle A, B\rangle = \langle B, A\rangle$

   (f) $\langle rA, B\rangle = r\langle A, B\rangle$

   (g) $\langle A + B, C\rangle = \langle A, C\rangle + \langle B, C\rangle$

   (h) $\|rA\| = |r| \cdot \|A\|$

   (i) $\|A\| > 0$ iff $A \neq (0, 0)$

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3. Consider the Euclidean plane $\mathscr{E} = (\mathbb{R}^2, \mathscr{L}_E, d_E)$. Let $A, B \in \mathbb{R}^2$. Prove that $d_E(A, B) = \|A - B\|$.
