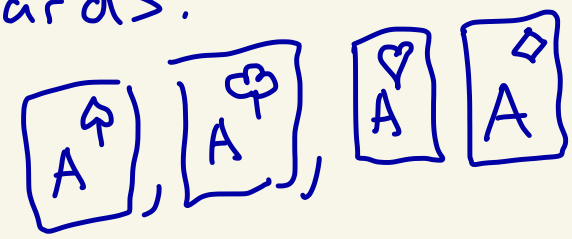
HW 2
Extra problems

(1)

There are $\binom{52}{2} = \frac{52!}{2!50!} = \frac{52 \cdot 51}{2} = 1326$ ways to be dealt two cards.

(a) There are four aces:  , Thus there are $\binom{4}{2}$ possible ways to be dealt two aces.

$$\binom{4}{2} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

Thus the probability of such an event is

$$\frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} \approx 0.00452489\ldots$$

or $\approx 0.45\%$

(b)

There are 13 possible face values:

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

Each face value has 4 suits.

Thus, there are

$$13 \cdot \binom{4}{2} = 13 \cdot \frac{4!}{2!2!} = 13 \cdot 6 = 78$$

13: choose the face value

$\binom{4}{2}$: choose two of the 4 cards, ie choose two from ♡, ◇, ♤, ♧

ways to get two cards of the same face value.

Thus, the probability of such an event is

$$\frac{13 \cdot \binom{4}{2}}{\binom{52}{2}} = \frac{78}{1326} \approx 0.5882\ldots$$

or $\approx 5.88\%$

(2) There are $6^4 = 1296$ ways to roll a six-side die four times in a row.

Let E be the event that a 3 occurs at least once in those four rolls.

Lets instead calculate $P(\overline{E})$ where $\overline{E}$ is the event that a 3 does not occur even once in four rolls.

There are five numbers that aren't 3, they are 1, 2, 4, 5, 6.

Thus the number of ways a 3 does not occur in four rolls is $5 \cdot 5 \cdot 5 \cdot 5 = 5^4 = 625$

roll 1 roll 2 roll 3 roll 4

Thus, $P(\bar{E}) = \frac{625}{1296} \approx 0.48.$

So,

$$P(E) = 1 - P(\bar{E}) = 1 - \frac{625}{1296}$$

$$= \frac{1296 - 625}{1296}$$

$$= \frac{671}{1296}$$

$$\approx \boxed{0.517747\ldots}$$

or $\approx 51.77\%$