Math 4740

HW 2 Solutions

1 (a)

1–9	0–9	0–9	0–9	0–9	0–9
9 possibilities	10 possibilities	10	10	10	10

There are

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = \boxed{900{,}000}$$

six digit numbers

① (b) Instead we calculate all the six digit numbers <u>without</u> a 5 and subtract this from 1(a).

1–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5	0–9 not 5
8 possibilities	9 possibilities	9	9	9	9

There are

$$\underbrace{900{,}000}_{\text{\# six digit numbers}} - \underbrace{8\cdot 9\cdot 9\cdot 9\cdot 9\cdot 9}_{\text{\# six digit numbers without a 5}}$$

$$= 900{,}000 - 472{,}392$$

$$= \boxed{427{,}608}$$

six digit numbers without a 5.

②

A - Z	A - Z not same as spot 1	A - Z not same as spot 1 or 2	0 - 9	0 - 9 not same as spot 4	0 - 9 not same as spot 4 or 5
26 possibilities	25 possibilities	24 possibilities	10 possibilities	9 possibilities	8 possibilities

\# of license plates is

$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = \boxed{11{,}232{,}000}$$

③ (a) There are 5 letters so there are $5! = 120$ permutations.

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③ (b)

$\underline{\ a\ }$ $\underline{\ b/d/e\ }$ $\underline{\ b/d/e\ }$ $\underline{\ b/d/e\ }$ $\underline{\ c\ }$

- Spot 2: b/d/e — 3 possibilities
- Spot 3: b/d/e, not same as spot 2 — 2 possibilities
- Spot 4: b/d/e, not same as spot 2 or 3 — 1 possibility

There are $\boxed{3 \cdot 2 \cdot 1 = 6}$ possible permutations.

Here is the choice tree:

- a b _ _ c
  - a b d _ c — a b d e c
  - a b e _ c — a b e d c
- a d _ _ c
  - a d b _ c — a d b e c
  - a d e _ c — a d e b c
- a e _ _ c
  - a e b _ c — a e b d c
  - a e d _ c — a e d b c

④ You need to put 5 dashes and 3 dots into $5+3=8$ spots.

The number of possible messages of this type can be calculated by picking the 5 spots amongst the 8 total spots where the dashes go.

This can be done in

$$\binom{8}{5} = \frac{8!}{3!\,5!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{6}\cdot \cancel{5!}} = 56 \text{ ways.}$$

Example: _ _ _ _ _ _ _ _

Since the other spots have to be dots there is only one possible way to fill in the remaining spots with dots.

Thus, the answer is

$$56\cdot 1 = \boxed{56 \text{ possible messages}}$$

(5) (a) Some examples are

01121120

11111111

12000121

Let's count!

$$\underset{\substack{\uparrow\\ 3\\ \text{possibilities}}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}\quad \underset{\substack{\uparrow\\ 3}}{\underline{0/1/2}}$$

There are $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^8$

$= \boxed{6561 \text{ possible sequences}}$

5 (b)

Step 1:

Pick 4 spots from the 8 total spots where the 0's go. This can be done in $\binom{8}{4} = \frac{8!}{4!4!}$

$= 70$ ways

One of the 70 ways:

$\underline{0}\ \underline{0}\ \underline{\ \ }\ \underline{0}\ \underline{\ \ }\ \underline{0}\ \underline{\ \ }\ \underline{\ \ }$

Step 2:

Now there is no choice at this point, you must fill the remaining 4 spots with 1's. Thus, only 1 possibility at this step.

The above example becomes

$\underline{0}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{1}$

Answer:

Total # of sequences is $70 \cdot 1 = \boxed{70}$

5 (c)

Step 1:
Pick 3 spots from the 8 total spots to put the 0's. There are

$$\binom{8}{3} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{5!}\,\cancel{6}} = 56$$

ways to do this

Example possibility

0 _ 0 0 _ _ _ _

Step 2:
Pick 3 spots from the remaining 5 spots to put the 1's in. There are

$$\binom{5}{3} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot \cancel{3!}}{2\cdot \cancel{3!}} = 10$$

ways to do this

Example possibility

0 _ 0 0 1 1 _ 1

Step 3: Only 1 choice to make now: Fill the remaining spots with 2's

Example possibility

0 2 0 0 1 1 2 1

Answer: Total number of sequences is $56\cdot 10\cdot 1 = \boxed{560}$

6 (a) There are 13 books. Thus there are $13! = \boxed{6,227,020,800}$ ways to put them on the shelf

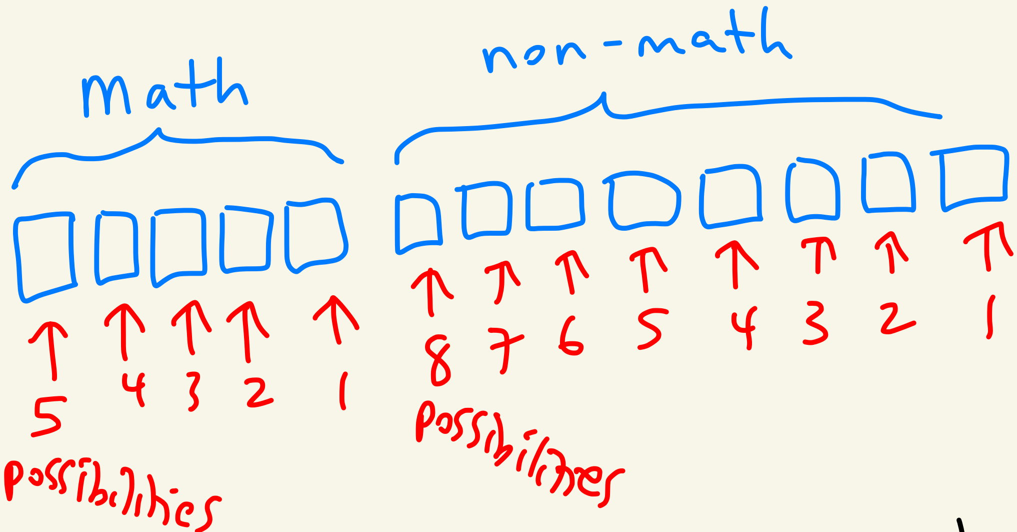
$$\underset{\substack{13 \\ \text{choices}}}{\underline{\quad}}\ \underset{12}{\underline{\quad}}\ \underset{11}{\underline{\quad}}\ \underset{\substack{10 \\ \text{choices}}}{\underline{\quad}}\ \underset{9}{\underline{\quad}}\ \underset{8}{\underline{\quad}}\ \underset{7}{\underline{\quad}}\ \underset{6}{\underline{\quad}}\ \underset{5}{\underline{\quad}}\ \underset{4}{\underline{\quad}}\ \underset{3}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{1}{\underline{\quad}}$$

⑥ (b) There are 5 math books and 8 other books. The math books have to be clumped together.

<u>Step 1:</u> Pick where the math books go. Think of the math books as one unit on this step

9 possibilities

Step 2: Now for each of the 9 possibilities from step 1 you must fill the books in in every possible permutation.



So step 2 gives

$$(5!)(8!) = (120)(40{,}320) = 4{,}838{,}400 \text{ ways}$$

Thus, the total # of ways to put the books on the shelf with the math books next to each other is

$$(9)(5!)(8!) = (9)(4{,}838{,}400) = \boxed{43{,}545{,}600 \text{ ways}}$$

7

There are $5+6=11$ people that can sit down in a row.

11	10	9	8	7	6	5	4	3	2	1

possibilities possibilities

So, there are $11! = 39{,}916{,}800$ ways the mathematicians and biologists can sit down.

Now we count the number of ways they can sit down with the mathematicians sitting together and the biologists sitting together.

Step 1: There are two possible templates.

M_1	M_2	M_3	M_4	M_5	B_1	B_2	B_3	B_4	B_5	B_6

OR

B_1	B_2	B_3	B_4	B_5	B_6	M_1	M_2	M_3	M_4	M_5

Step 2: Now fill in the spots.

M 1	M 2	M 3	M 4	M 5	B 1	B 2	B 3	B 4	B 5	B 6
5 possibilities	4	3	2	1	6 possibilities	5	4	3	2	1

This case gives $(5!)(6!)$ possibilities

OR

B 1	B 2	B 3	B 4	B 5	B 6	M 1	M 2	M 3	M 4	M 5
6 possibilities	5	4	3	2	1	5 possibilities	4	3	2	1

This case gives $(6!)(5!)$ possibilities

Adding these cases gives

$$(5!)(6!) + (6!)(5!) = 86{,}400 + 86{,}400$$

$$= 172{,}800 \text{ ways}$$

Thus the probability is

$$\boxed{\frac{172{,}800}{39{,}916{,}800} \approx 0.004329\ldots \approx 0.43\%}$$

8. The sample space size is $|S| = 6^6 = 46{,}656$.

Let E be the event that at least two of the dice have the same number. We want $P(E)$.

Instead we will calculate $P(E) = 1 - P(\overline{E})$ where $\overline{E}$ is the event that none of the dice have the same number.

die 1	die 2	die 3	die 4	die 5	die 6
6 choices	5 choices since can't be the same as die 1	4 choices since can't be the same as die 1 or die 2	3 choices since can't be the same as die 1 or die 2 or die 3	2 choices since can't be the same as die 1 or die 2 or die 3 or die 4	1 choice since can't be the same as die 1 or die 2 or die 3 or die 4 or die 5

So, $$P(\overline{E}) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6^6} = \frac{720}{46{,}656}$$

Thus, $$P(E) = 1 - P(\overline{E}) = \frac{46{,}656 - 720}{46{,}656} = \boxed{\frac{45{,}936}{46{,}656}}$$

$$\approx \boxed{0.9845679\ldots}$$

$$\approx \boxed{98.46\%}$$

9

(a) $\underbrace{\binom{10}{5}}_{\substack{\text{choose}\\5\\\text{Biologists}}}\underbrace{\binom{8}{4}}_{\substack{\text{Choose 4}\\\text{Mathematicians}}} = \frac{10!}{5!\,5!}\cdot\frac{8!}{4!\,4!}$

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}\;5!}\cdot\frac{8\cdot 7\cdot 6\cdot 5\cdot \cancel{4!}}{4!\;\cancel{4!}}$$

$$= \frac{30{,}240}{120}\cdot\frac{1{,}680}{24}$$

$$= 252\cdot 70 = \boxed{17{,}640 \text{ possible committees}}$$

(b) Suppose mathematician A and mathematician B don't want to serve together. Let's count the answer indirectly by counting the number of committees that both mathematicians A and B are on and subtract that from the total number of committees.

Pick 5 Biologists

from the remaining 6 Mathematicians choose 2 more

$$\overbrace{\binom{10}{5}}^{\text{Pick 5 Biologists}} \underbrace{\binom{2}{2}}_{\text{choose both mathematicians A and B}} \overbrace{\binom{6}{2}}^{\text{from the remaining 6 Mathematicians choose 2 more}} = \left[\frac{10!}{5!\,5!}\right]\cdot\left[1\right]\cdot\left[\frac{6!}{2!\,4!}\right]$$

choose both mathematicians A and B

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{5!\,\cancel{5!}}\cdot\frac{6\cdot 5\cdot \cancel{4!}}{2!\,\cancel{4!}}$$

$$= \frac{30{,}240}{120}\cdot\frac{30}{2} = 252\cdot 15$$

$$= 3{,}780$$

So there are 3,780 committees that mathematician A and B are both serving on.

Thus, there are

$17{,}640 - 3{,}780 =$ $\boxed{13{,}860 \text{ committees}}$

that don't have both mathematician A and B on them.

There is another direct way to calculate (b).
Let's see how to do this.
We break the count into parts.

$$\left[\begin{array}{l}\text{\# of committees with}\\ \text{mathematician A but}\\ \text{not mathematician B}\end{array}\right] = \underbrace{\binom{10}{5}}_{\substack{\text{pick 5}\\ \text{Biologists}}} \overbrace{\binom{1}{1}}^{\substack{\text{pick}\\ \text{mathematician A}}} \underbrace{\binom{6}{3}}_{\substack{\text{pick 3 more}\\ \text{mathematicians}\\ \text{that aren't}\\ \text{mathematician}\\ \text{A or B}}} = 5{,}040$$

$$\left[\begin{array}{l}\text{\# of committees with}\\ \text{mathematician B but}\\ \text{not mathematician A}\end{array}\right] = \underbrace{\binom{10}{5}}_{\substack{\text{pick 5}\\ \text{Biologists}}} \overbrace{\binom{1}{1}}^{\substack{\text{pick}\\ \text{mathematician B}}} \underbrace{\binom{6}{3}}_{\substack{\text{pick 3 more}\\ \text{mathematicians}\\ \text{that aren't}\\ \text{mathematician}\\ \text{A or B}}} = 5{,}040$$

$$\left[\begin{array}{l}\text{\# of committees with}\\ \text{neither mathematician}\\ \text{A nor B}\end{array}\right] = \underbrace{\binom{10}{5}}_{\substack{\text{pick 5}\\ \text{biologists}}} \underbrace{\binom{6}{4}}_{\substack{\text{pick 4 mathematicians}\\ \text{that aren't mathematician}\\ \text{A nor B}}} = 3{,}780$$

Adding these three numbers up gives us the number of committees where mathematician A and B aren't serving together: $5{,}040 + 5{,}040 + 3{,}780 = \boxed{13{,}860}$

(10) The sample space has size

$|S| = 8 \cdot 8 \cdot 8 \cdot 8 = 8^4 = 4{,}096.$

(a) First choose where the two 3's can go by picking two of the four spots.

$$\boxed{\binom{4}{2} = \frac{4!}{2!\,2!} = 6}$$

$$\left.\begin{array}{cccc}
\underline{3} & \underline{3} & \underline{\ \ } & \underline{\ \ } \\
\underline{3} & \underline{\ \ } & \underline{3} & \underline{\ \ } \\
\underline{3} & \underline{\ \ } & \underline{\ \ } & \underline{3} \\
\underline{\ \ } & \underline{3} & \underline{3} & \underline{\ \ } \\
\underline{\ \ } & \underline{3} & \underline{\ \ } & \underline{3} \\
\underline{\ \ } & \underline{\ \ } & \underline{3} & \underline{3}
\end{array}\right\} \text{these are the 6 possibilities we are counting}$$

Then fill in the remaining two spots with two numbers that aren't 3's.

ex: $\underline{3}\ \ \underline{3}\ \ \underset{\substack{\uparrow \\ 7 \\ \text{choices}}}{\underline{\ \ \ }}\ \ \underset{\substack{\uparrow \\ 7 \\ \text{choices}}}{\underline{\ \ \ }}$

$$\boxed{7 \cdot 7 = 49}$$

There are $6 \cdot 49 = 294$ possibilities.

So, the probability is $\boxed{\dfrac{294}{4096}} \approx \boxed{0.07178\ldots}$

$\approx \boxed{7.18\,\%}$

(b)

$$P(\text{at most two 8's}) = P(\text{no 8's}) + P(\text{exactly one 8}) + P(\text{exactly two 8's})$$

$$= \frac{\overbrace{7\cdot 7\cdot 7\cdot 7}^{\text{fill in the four spots with \#s that aren't 8's}}}{4096}$$

$$+ \frac{\overbrace{\binom{4}{1}\cdot 7\cdot 7\cdot 7}^{\text{choose where the one 8 goes then fill in the remaining three spots with \#s that aren't 8's}}}{4096}$$

note:

$\binom{4}{1}=4$ is counting these 4 possibilities

8 _ _ _

_ 8 _ _

_ _ 8 _

_ _ _ 8

$$+ \frac{\overbrace{\binom{4}{2}\cdot 7\cdot 7}^{\text{choose where the two 8's go and then fill in the remaining two spots with \#s that aren't 8's}}}{4096}$$

Note

$\binom{4}{1}=4$

$\binom{4}{2}=6$

$$= \frac{2401+1372+294}{4096} = \boxed{\frac{4067}{4096}} \approx \boxed{0.9929\ldots} \approx \boxed{99.3\,\%}$$

(c)

$$P(\text{at least three 1's}) = P(\text{exactly three 1's}) + P(\text{exactly four 1's})$$

$$= \frac{\overbrace{\binom{4}{3}\cdot 7}^{\substack{\text{pick 3 spots} \\ \text{out of the 4 spots} \\ \text{for the 1's. Then} \\ \text{fill the remaining spot} \\ \text{with \#s that aren't 1.}}}}{4096} + \frac{\overbrace{\binom{4}{4}}^{\substack{\text{fill in all} \\ \text{4 spots with 1's} \\ \text{there is only one way} \\ \text{to do this}}}}{4096}$$

$$= \frac{4\cdot 7}{4096} + \frac{1}{4096} = \boxed{\frac{29}{4096}} \approx \boxed{0.00708}$$

$$\approx \boxed{0.7\%}$$

11

The sample space has size $6^{10} = 60{,}466{,}176$

Now count possibilities.

example possibility at this step

Step 1: Pick where the one 4 goes.

$\binom{10}{1} = 10$ possibilities

_ _ _ 4 _ _ _ _ _ _

example possibility at this step

Step 2: Pick where the six 5's go.

$$\binom{9}{6} = \frac{9!}{6!\,3!} = \frac{9\cdot 8\cdot 7\cdot 6!}{6!\,3!} = \frac{9\cdot 8\cdot 7}{3!} = 84 \text{ possibilities}$$

_ 5 5 4 5 _ 5 _ 5 5

example possibility at this step

Step 3: Fill in the other three spots with numbers that aren't 4 or 5.

$4\cdot 4\cdot 4 = 64$ possibilities

1 5 5 4 5 1 5 2 5 5

4 choices   4 choices   4 choice

The probability is thus

$$\frac{(10)(84)(64)}{60{,}466{,}176} = \boxed{\frac{53{,}760}{60{,}466{,}176}}$$

$$\approx \boxed{0.000889\ldots}$$

$$\approx \boxed{0.0889\,\%}$$

(12) The sample space has size $|S| = 2^5 = 32$

(a) Pick where the one head goes: $\binom{5}{1} = 5$

Fill in the remaining 4 spots with tails: $1 \cdot 1 \cdot 1 \cdot 1 = 1$

$$P(\text{exactly one head}) = \frac{5 \cdot 1}{32}$$

$$= \boxed{\frac{5}{32}} \approx \boxed{0.15625\ldots}$$

$$\approx \boxed{15.6\%}$$

We counted these possibilities:

H	T	T	T	T
T	H	T	T	T
T	T	H	T	T
T	T	T	H	T
T	T	T	T	H

(b) Pick where the three heads go: $\binom{5}{3} = \frac{5!}{3!2!} = 10$

Fill in the remaining 2 spots with tails: $1 \cdot 1 = 1$

$$P(\text{exactly three heads}) = \boxed{\frac{10}{32}}$$

$$\approx \boxed{0.3125\ldots} \approx \boxed{31.25\,\%}$$

Note: The count of 10 above counted these:

H H H T T
H H T H T
H H T T H
H T H H T
H T H T H
H T T H H
T H H H T
T H H T H
T H T H H
T T H H H

(c) There is only 1 way to get all tails. It is $\underline{T}\ \underline{T}\ \underline{T}\ \underline{T}\ \underline{T}$

So,

$$p(\text{all tails}) = \boxed{\frac{1}{32}} \approx \boxed{0.03125} \approx \boxed{3.125\%}$$

(13) The sample space has size

$$|S| = 2^{20} = 1{,}048{,}576$$

(a)

$$P(\text{at least 2 heads}) = 1 - P(\text{less than 2 heads})$$

$$= 1 - P(\text{exactly 0 heads}) - P(\text{exactly 1 head})$$

only 1 way to have 0 heads. Fill all 20 spots with tails

exactly 1 head. Pick the spot where the head goes in $\binom{20}{1} = 20$ ways. Then fill the remaining spots with tails in 1 way.

$$= 1 - \frac{\overbrace{1}}{1{,}048{,}576} - \frac{\overbrace{\binom{20}{1}}}{1{,}048{,}576}$$

$$= \frac{1{,}048{,}576 - 1 - 20}{1{,}048{,}576} = \boxed{\frac{1{,}048{,}555}{1{,}048{,}576}}$$

$$\approx \boxed{0.99997997\ldots}$$

$$\approx \boxed{99.998\%}$$

(b)

$$P(\text{at most 3 heads}) = P(\text{0 heads}) + P(\text{exactly 1 head}) + P(\text{exactly 2 heads}) + P(\text{exactly 3 heads})$$

$$= \frac{1}{2^{20}} + \frac{\overbrace{\binom{20}{1}}^{\text{pick 1 spot out of 20 for the head then fill the rest with tails in 1 way}}}{2^{20}} + \frac{\overbrace{\binom{20}{2}}^{\text{pick 2 spots out of 20 for the heads then fill the rest with tails in 1 way}}}{2^{20}} + \frac{\overbrace{\binom{20}{3}}^{\text{pick 3 spots out of 20 for the heads then fill the rest with tails in 1 way}}}{2^{20}}$$

$$= \frac{1+20+190+1140}{1{,}048{,}576} = \boxed{\frac{1{,}351}{1{,}048{,}576}}$$

$$\approx \boxed{0.00128841} \approx \boxed{0.1288\ \%}$$

(14)

The sample space has size

$$|S| = \binom{20}{5} = \frac{20!}{5!\,15!} = \frac{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot \cancel{15!}}{5!\,\cancel{15!}}$$

$$= \frac{1{,}860{,}480}{120} = 15{,}504$$

To count how many ways we can pick 5 numbers so the smallest number is larger than 6 we must pick 5 numbers from the 14 circled below.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

This can be done in $\binom{14}{5} = \frac{14!}{5!\,9!} = \frac{14\cdot 13\cdot 12\cdot 11\cdot 10\cdot \cancel{9!}}{5!\,\cancel{9!}}$

$$= \frac{240{,}240}{120} = 2{,}002$$

Thus the probability is

$$\boxed{\frac{2{,}002}{15{,}504}} \approx \boxed{0.129} \approx \boxed{12.9\,\%}$$

(15) Recall there are

$\binom{47}{5} \cdot 27 = 41{,}416{,}353$ possible tickets

(a) The number of tickets that get 2 of the 5 lucky #s correct and the mega number is

$$\frac{\overbrace{\binom{5}{2}}^{\text{pick 2 of the 5 winning lucky numbers}} \cdot \overbrace{\binom{42}{3}}^{\text{pick 3 non-winning lucky numbers}} \cdot \overbrace{\binom{1}{1}}^{\text{pick the winning mega number}}}{41{,}416{,}353} = \frac{(10)(11{,}480)}{41{,}416{,}353} = \boxed{\frac{114{,}800}{41{,}416{,}353}}$$

$$\approx \boxed{0.00277\ldots}$$

$$\approx \boxed{0.277\ \%}$$

(b) The number of tickets that get 4 of the 5 lucky #s correct and the mega number is

$$\frac{\underbrace{\binom{5}{4}}_{\substack{\text{pick 4 of the 5 winning}\\\text{lucky numbers}}} \cdot \underbrace{\binom{42}{1}}_{\substack{\text{pick 1 non-winning}\\\text{luck number}}} \cdot \underbrace{\binom{1}{1}}_{\substack{\text{pick the winning}\\\text{mega number}}}}{41{,}416{,}353} = \frac{(5)(42)}{41{,}416{,}353} = \boxed{\frac{210}{41{,}416{,}353}}$$

$$\approx \boxed{0.00000507\ldots}$$

$$\approx \boxed{0.000507\ \%}$$

(16) There are 49 remaining cards. Thus, there are $\binom{49}{2} = \frac{49!}{2!\,47!} = \frac{49 \cdot 48 \cdot 47!}{2!\,47!} = \frac{49 \cdot 48}{2} = 1{,}176$ possible two card combinations that you can get.

(a) There are $13 - 3 = 10$ remaining clubs. So, the odds of getting two clubs is

$$\frac{\binom{10}{2}}{\binom{49}{2}} = \boxed{\frac{45}{1{,}176}} \approx \boxed{0.038\ldots} \approx \boxed{3.8\ \%}$$

(b) The cards that give you a straight are

$\underbrace{\boxed{A^?}\ \boxed{5^?}}_{\text{gives}}$ or $\underbrace{\boxed{5^?}\ \boxed{6^?}}_{\text{gives}}$

$\boxed{A^?}\ \boxed{2^♣}\ \boxed{3^♣}\ \boxed{4^♣}\ \boxed{5^?}$ $\qquad$ $\boxed{2^♣}\ \boxed{3^♣}\ \boxed{4^♣}\ \boxed{5^?}\ \boxed{6^?}$

where ? is any suit except you don't want to count $\boxed{A^♣}\ \boxed{5^♣}$ or $\boxed{5^♣}\ \boxed{6^♣}$ since those would give you a straight flush.

↓

Thus, the number of hands that give you a straight but not a straight flush is

$$\underbrace{4\cdot 4}_{\boxed{A^{?}}\ \boxed{5^{?}}} + \underbrace{4\cdot 4}_{\boxed{5^{?}}\ \boxed{6^{?}}} - \underbrace{2}_{\substack{\boxed{A^{\clubsuit}}\ \boxed{5^{\clubsuit}} \\ \boxed{5^{\clubsuit}}\ \boxed{6^{\clubsuit}}}} = 30$$

So, the probability is $\boxed{\dfrac{30}{1,176}} \approx \boxed{0,02551} \approx \boxed{2.551\ \%}$

(c) The cards that give you a straight flush are $\boxed{A^{\clubsuit}}\ \boxed{5^{\clubsuit}}$ and $\boxed{5^{\clubsuit}}\ \boxed{6^{\clubsuit}}$. Thus, the probability is $\boxed{\dfrac{2}{1,176}} \approx \boxed{0.0017\ldots} \approx \boxed{0.17\ \%}$