Math 3450 - Homework # 2
Set Theory

1. Let $A = \{1, 5, -12, 100, 1/3, \pi\}$, $B = \{5, 1, -12, 18, -1/3\}$, $C = \{10, -1, 0\}$, $D = \{1, 2\}$, and $E = \{1, -1\}$. Calculate the following:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A \cap C$
   (d) $A \cap \emptyset$
   (e) $B \cup \emptyset$
   (f) $D \times E$
   (g) $(D \cap A) \times (E \cup D)$
   (h) $C \times D$
   (i) $A - B$
   (j) $C - A$
   (k) $A - \emptyset$
2. Let $A = \{2k \,|\, k \in \mathbb{Z}\}$ and $B = \{3n \,|\, n \in \mathbb{Z}\}$. Prove that $A \cap B = \{6m \,|\, m \in \mathbb{Z}\}$.
3. Let A, B, and C be sets. Prove that if $A \subseteq B$, then $A - C \subseteq B - C$.
4. Let A and B be sets. Prove that $A \subseteq B$ if and only if $A - B = \emptyset$.
5. Let A, B, and C be sets. Prove that if $A \subseteq B$, then $A \cup C \subseteq B \cup C$.
6. Let A, B, and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
7. Let A, B, and C be sets. Prove or disprove: If $A \cap B \neq \emptyset$ and $B \cap C \neq \emptyset$, then $A \cap C \neq \emptyset$.
8. Let $A_n = \{x \in \mathbb{Z} \,|\, -n \leq x \leq n\}$. List the elements in the sets A_1, A_2, A_3, and A_4. Then calculate the following sets $\bigcap_{i=2}^{\infty} A_n$ and $\bigcup_{i=5}^{\infty} A_n$.
9. Calculate the following intersections and unions.
   (a) Calculate $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$ where $A_n = (-n, n)$.

(b) Calculate $\bigcup_{n=2}^{\infty} A_n$ and $\bigcap_{n=2}^{\infty} A_n$ where $A_n = (1/n, 1)$.

(c) Calculate $\bigcup_{n=3}^{\infty} A_n$ and $\bigcap_{n=3}^{\infty} A_n$ where $A_n = (2 + 1/n, n)$.

10. Let A, B, and C be sets. Prove that $A \cap (B \cap C) = (A \cap B) \cap C$.

11. Let A, B, and C be sets. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

12. Let A, B, and C be sets. Prove that if $A \subseteq B$ then $A \subseteq B \cup C$.

13. Let $A = \{1, x, 5\}$. List the elements of the power set $\mathcal{P}(A)$.

14. Let A and B be sets.

    (a) Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

    (b) Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

    (c) Give an example where $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$.

15. Let A and B be sets. Prove that $A \setminus B$ and B are disjoint.

16. Let A, B, C, and D be sets. Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.