

Math 474 - Homework # 1

Sets and Probability Spaces

1. (a) Consider the experiment of rolling two 6-sided dice. What is a sample space S and a probability function P for this experiment? (b) Let A be the event that the sum of the dice is 2. Let B be the event that the sum of the dice is 4. What are the elements of A and B ? (c) Calculate $A \cup B$, $A \cap B$, \overline{A} , \overline{B} . (d) How do the sets from part c relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events from parts b and c.
2. Consider the experiment of rolling two 6-sided dice. (a) What is a sample space S and a probability function P for this experiment? (b) Let A be the event that at least one of the dice is even. Let B be the event that both of the dice are even. Let C be the event that at least one of the dice is odd. Let D be the event that both of the dice are odd. List the elements of A , B , C , and D . (c) Calculate $A \cap C$, \overline{A} , $B \cap D$, $B \cup D$, \overline{B} , \overline{D} . (d) How do the sets from part c relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.
3. Consider the experiment of flipping a coin 4 times in a row. (a) What is a sample space S and a probability function P for this experiment? (b) Let A be the event that a head occurs on the first flip. Write out the elements of A . Let B be the event that a tails occurs on the second flip and the fourth flip. Write out the elements of B . (c) Calculate $A \cup B$, $A \cap B$, \overline{A} , \overline{B} . (d) How do these sets relate to logic? What logical sentences express them? (e) Calculate the probabilities of the events in parts b and c.
4. Consider the experiment of rolling a weighted 4-sided dice. Weighted means that the probabilities aren't equal for each number on the dice. Suppose that through experimentation you discover that the 1 occurs two times for every eight rolls of the dice, a 2 occurs two times for every eight rolls of the dice, a 3 occurs three times for every eight rolls of the dice, and a 4 occurs one time for every eight rolls of the dice. (a) What is a sample space S and a probability function P for this experiment?

- (b) Let A be the event of rolling a 1 or a 3. What are the elements of A and what is the probability of A ? (c) Let B be the event of rolling a 1 or a 2 or a 3. What are the elements of B and what is the probability of B ?
5. Suppose that two 4-sided dice are thrown.
 - (a) What is the probability that at least one of the dice shows a 2?
 - (b) What is the probability that the sum of the dice is 4?
 - (c) What is the probability that the sum of the dice is either a 5 or a 7?
 6. Suppose that two 8-sided dice are thrown, one green and one red. What is the probability that the red die has a larger value than the green die?
 7. Consider an bag that contains three balls: 1 white balls, 1 red ball, and 1 green ball. Suppose you want to construct a probabilistic model of the experiment where you randomly (without looking) choose 1 ball from the bag and record the result. How could you do such a thing? What is the sample space S ? Describe the probability function P . What is the probability of choosing a white ball? Of choosing a red ball?
 8. Suppose we take the same setup as problem 7. But we change it to the experiment where you randomly choose 2 balls from the bag. What is a possible sample space S and probability function P for this model? What is the probability of choosing one white ball and one red ball?
 9. Consider the experiment where one 4-sided dice is thrown continually until a 2 is rolled. Construct a probability space that models this experiment. Verify that you have a probability space.
 10. Consider the experiment where a coin is flipped continually until a head occurs. Construct a probability space that models this experiment. Verify that you have a probability space.