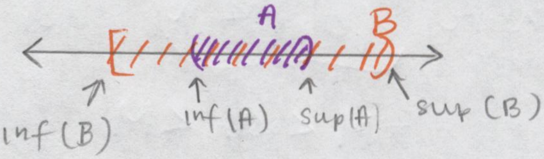
P.1 9/10 Monday Week 4

HW 1 # 6 Let A and B be subsets of $\mathbb{R}$. Suppose A and B are bounded from above and below. Suppose

Suppose $A \subseteq B$



(a) Prove $\sup(A) \le \sup(B)$ and $\inf(B) \le \inf(A)$

proof: Let $S_A = \sup(A)$ and $S_B = \sup(B)$

By def of supremum,

- $a \le S_A \ \forall a \in A$
- $S_A \le c$ for any other upper-bounds c of A

since S_B is the supremum of B

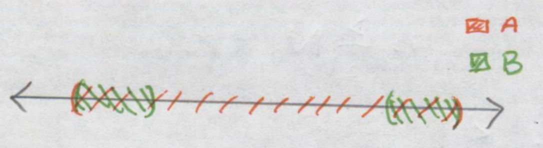
$$b \le S_B \ \forall b \in B$$

Since $A \subseteq B$, $a \le S_B \ \forall a \in A$. So, S_B is an upper bound of A. So $S_A \le S_B$ (Do inf part as practice)

(b) If $\sup(A) = \sup(B)$ and $\inf(A) = \inf(B)$ do we necessarily have $A = B$?

No! Let $A = (0, 10)$

$B = (0, 2) \cup (8, 10)$



counter-example $\sup(A) = \sup(B) = 10$

$\inf(A) = \inf(B) = 0$ but $A \neq B$

another example: $A = (-2, 5)$ but $A \neq B$

$B = [-2, 5]$