The Completeness Axiom

HW 1

Def: Let $S \subseteq \mathbb{R}$, where $S \neq \emptyset$.

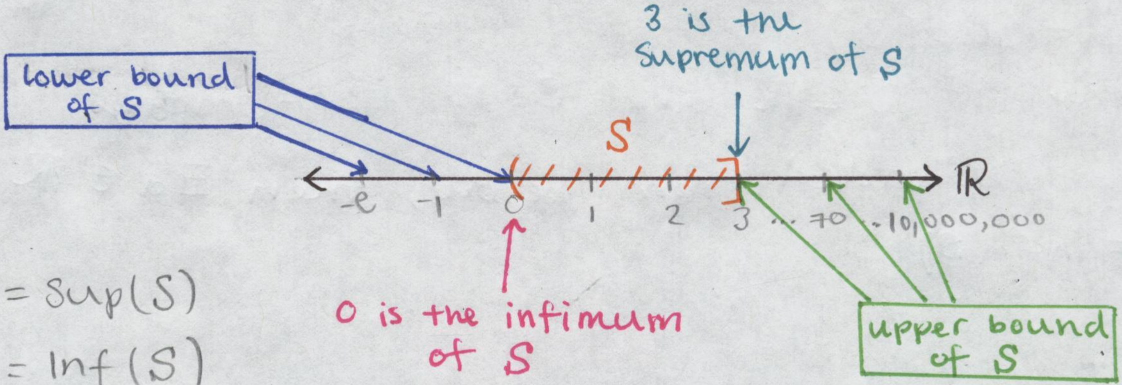
- We say that S is **bounded from above** if there exists $b \in \mathbb{R}$ where $x \leq b$ for all $x \in S$. If this is the case then we call b an **upper bound** for S.

  Furthermore, if b is an upper bound for S and $b \leq c$ for all upper bounds of S, then we call b the **least upper bound** or **supremum** of S, and we write $b = \sup(S)$.

- We say that S is **bounded from below** if there exists $a \in \mathbb{R}$ where $a \leq x$ for all $x \in S$. If this is the case we call a a **lower bound** for S.

  Furthermore, if a is a lower bound for S and $c \leq a$ for all lower bounds of S, then we call a the **greatest lower bound** or **infimum** of S, and we write $a = \inf(S)$.

Example: $S = (0, 3]$



$3 = \sup(S)$

$0 = \inf(S)$