

### Significant Figures

When writing a number, it is important to know how well the value of the number is known.

#### Examples

If I say I have 12 donuts, I usually mean I have exactly 12 donuts because donuts do not come in fractions of a unit.

### Significant Figures

If I say I have 12 grams of salt, does that mean exactly 12 grams or something between 11.5 and 12.5 grams.

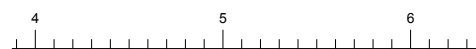
Because the mass of an object in grams may be expressed in fractions of grams, I must specify as precisely as possible what the mass is.

Significant figures indicate the precision with which a number has been determined.

### Significant Figures

When expressing a value, the number of digits used indicates the number of digits that could be measured accurately.

The final digit in the value is an estimate of the least precise number.



Length = 5.73 units

### Significant Figures

The manner in which a number is written indicates the number of significant figures it has.

#### Examples

4000      1 significant figure

the zeros are simply place holders and do not indicate precision of the measurement.

in scientific notation:  $4 \times 10^3$

### Significant Figures

4000.      4 significant figures

By placing a decimal point after the last zero, this indicates that the zeros were measured

in scientific notation:  $4.000 \times 10^3$

### Significant Figures

4000.000      7 significant figures

zeros following the decimal point with no other digits behind them also indicate precision of measurement, so they count as significant figures

### Significant Figures

.0004      1 significant figure  
 for numbers less than one, zeros following the decimal point, but before the first digit, are simply place holders and do not indicate precision  
 in scientific notation:  $4 \times 10^{-4}$

### Significant Figures

.000400    3 significant figures  
 zeros following digits in numbers less than 1 indicate precision and are significant.  
 in scientific notation:  $4.00 \times 10^{-4}$

### Mathematical Rules for Significant Figures

#### Addition/Subtraction

When adding or subtracting numbers, the number of significant figures in the result is determined from the position relative to the decimal point of the least significant figure of the numbers being added or subtracted

### Mathematical Rules for Significant Figures

Add 472.1, 3.192, and 5000.86

$$\begin{array}{r} 472.1 \\ 3.192 \\ + 5000.86 \\ \hline 5476.152 \end{array}$$

Because 472.1 has only 1 sig. fig. after the decimal point, the final answer can have only 1 sig. fig. after the decimal point—the correct answer is **5476.2**

### Mathematical Rules for Significant Figures

Subtract 126.5419 from 8000:

$$\begin{array}{r} 8000 \\ - 126.5419 \\ \hline 7873.4581 \end{array}$$

Because 8000 has only 1 sig. fig. four places to the left of the decimal point, the least significant figure in the final answer must also be four places to the left of the decimal point—correct answer is **8000**

### Mathematical Rules for Significant Figures

#### Multiplication/Division

The number of significant figures in the result is determined from the number of significant figures in the least significant value used in the calculation

### Mathematical Rules for Significant Figures

Multiply 88.037 by .00721

$$\begin{array}{r} 88.037 \\ \times .00721 \\ \hline 0.63474677 \end{array}$$

88.037 has 5 sig. figs.  
and .00721 has 3 sig.  
figs. This limits the result  
to a total of three sig. figs.  
—the correct answer is  
0.635

### Mathematical Rules for Significant Figures

#### Logarithms/Exponentiation

Remember the definitions of logarithms:

$$x = 10^y \quad \log(x) = y$$

$$1000 = 10^3 \quad \log(1000) = 3$$

### Mathematical Rules for Significant Figures

#### Logarithms/Exponentiation

Logarithms have a *characteristic* and a *mantissa*:

$$\log(87.21) = 1.9406$$

characteristic      mantissa

The *characteristic* determines the appropriate power of ten for the result, and the *mantissa* relates to the proper value of the number

### Mathematical Rules for Significant Figures

#### Logarithms/Exponentiation

The correct number of significant figures for a logarithmic operation is determined by the number of significant figures in the mantissa

### Mathematical Rules for Significant Figures

#### Logarithms/Exponentiation

Example:

$$\log(2.18 \times 10^{-6}) = -5.662$$

the original number has 3 sig. figs., so the resulting mantissa should also have 3 sig. figs., and the result has 4 sig. figs.

-5, the characteristic, indicates the number of 0s serving as place holders

### Mathematical Rules for Significant Figures

#### Logarithms/Exponentiation

Example:

$$10^{4.5} = 3 \times 10^4$$

the mantissa of 4.5 has only 1 sig. fig.

the characteristic = 4 and indicates the power of ten in the result

the result will have only 1 sig. fig.

### Accuracy and Precision

Accuracy and precision have very different meaning in quantitative analysis

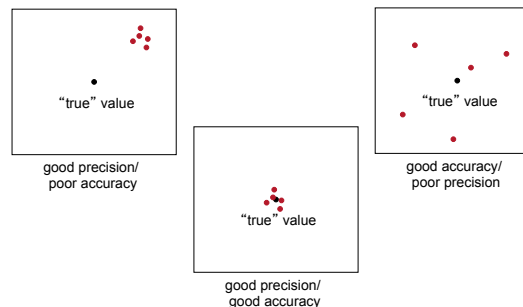
#### Definitions

Accuracy—how well a result represents the “true” value for a measurement

Precision—how well a set of measurements can be repeated with the same result

Ideally, we want both accuracy and precision in our lab work

### Accuracy and Precision



### Types of Experimental Errors

#### Determinate (systematic) errors

Determinate errors are errors that result from a flaw in the experimental procedure

- miscalibrated instrument
- operator mistakes

Determinate errors occur the same way every time an analysis is made

They can be corrected by calibrating instruments or training operators

### Types of Experimental Errors

#### Random errors

Random errors result from the physical variability of measurements

Random errors have an equal chance of being either positive or negative

- noise in instruments
- uncertainty in making readings

### Uncertainty and Errors

If we assume that determinate errors have been eliminated, the uncertainty in any quantitative result will come from random errors

### Uncertainty and Errors

The uncertainty of a single measurement can be determined either:

- ◆ from the equipment used
  - a buret has an uncertainty of  $\pm 0.02$  mL
  - an analytical balance has an uncertainty of approximately  $\pm 0.0003$  g

### Uncertainty and Errors

The uncertainty of a single measurement can be determined either:

- ♦ performing repeated measurements and calculating the standard deviation when using an atomic absorption spectrometer, measure the absorbance of a sample five times

### Uncertainty and Errors

Uncertainty in a result can be expressed as either absolute uncertainty (e) or relative uncertainty (%e) (for now, we will use e to represent the uncertainty in a quantity)

#### Absolute uncertainty

$$m = 3.125 \pm 0.037 \text{ g}$$

$$e = 0.037 \text{ g is uncertainty in mass}$$

absolute uncertainty has same units as the result

### Uncertainty and Errors

#### Relative uncertainty

Relative uncertainty is calculated by dividing the absolute uncertainty by the result and multiplying by 100%

$$m = 3.125 \pm 0.037 \text{ g}$$

$$\%e = \frac{0.037 \text{ g}}{3.125 \text{ g}} \times 100\% = 1.18\%$$

$$m = 3.125 \text{ g} \pm 1.18\%$$

relative uncertainty is always expressed as a percentage

subscripted 8 indicates this digit is insignificant, but we will keep until a final answer is determined

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Addition/Subtraction—use absolute uncertainties

$$X_{\text{final}} = X_1 + X_2 + X_3$$

$$X_1 \pm e_1 \quad X_2 \pm e_2 \quad X_3 \pm e_3$$

$$e_{\text{final}} = \sqrt{e_1^2 + e_2^2 + e_3^2} \quad X_{\text{final}} \pm e_{\text{final}}$$

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Addition/Subtraction

$$X_{\text{final}} = X_1 - X_2 + X_3$$

$$x_1 = 1.982 \pm 0.008 \quad x_2 = 0.752 \pm 0.052 \quad x_3 = 25.154 \pm .853$$

$$e_{\text{final}} = \sqrt{(.008)^2 + (.052)^2 + (.853)^2} = 0.854$$

$$x_{\text{final}} = 26.384 \pm 0.854$$

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

Multiplication/Division—use relative uncertainties

$$X_{\text{final}} = (X_1)(X_2)(X_3)$$

$$X_1 \pm \%e_1 \quad X_2 \pm \%e_2 \quad X_3 \pm \%e_3$$

$$e_{\text{final}} = \sqrt{\%e_1^2 + \%e_2^2 + \%e_3^2} \quad X_{\text{final}} \pm \%e_{\text{final}}$$

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

#### Multiplication/Division

$$x_{\text{final}} = \frac{(x_1)(x_2)}{x_3}$$

$$x_1 = 2.341 \pm 1.12\% \quad x_2 = 8.34 \times 10^{-4} \pm 3.83\% \quad x_3 = 1280 \pm 0.239\%$$

$$e_{\text{final}} = \sqrt{(1.12)^2 + (3.83)^2 + (239)^2} = 3.99_8\%$$

$$x_{\text{final}} = 1.52_5 \times 10^{-6} \pm 3.99_8\%$$

$$x_{\text{final}} = (1.52_5 \pm 0.06_1) \times 10^{-6}$$

### Uncertainty and Errors

When performing calculation involving mixed operations (addition/subtraction and multiplication/division)

1. Perform addition/subtraction, determine absolute error of result, and then relative error of result
2. Perform multiplication/division operation using relative errors

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

#### Exponentiation and Logarithms

$$\text{For } y = x^a \quad x \pm \%e_x$$

$$\%e_y = a(\%e_x)$$

$$y = x^3 \quad x = 5.981 \pm 2.13\%$$

$$\%e_y = 3(2.13\%) = 6.39\%$$

$$y = 214.0 \pm 6.39\%$$

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

#### Exponentiation and Logarithms

$$\text{For } y = \log(x) \quad x \pm e_x$$

$$e_y = \frac{1}{\ln 10} \frac{e_x}{x}$$

$$y = \log(x) \quad x = 3.17 \pm 0.28$$

$$e_y = (.43429) \cdot (0.28) / (3.17) = 0.038_4$$

$$y = 0.501 \pm 0.038$$

### Uncertainty and Errors

The uncertainty of a calculated result is determined depending on the calculations performed:

#### Exponentiation and Logarithms

$$\text{For } y = \ln(x) \quad x \pm e_x$$

$$e_y = \frac{e_x}{x}$$

$$y = \ln(x) \quad x = 7.432 \times 10^{-5} \pm 0.917 \times 10^{-5}$$

$$e_y = (0.917 \times 10^{-5}) / (7.432 \times 10^{-5}) = 0.123_3$$

$$y = -9.507 \pm 0.123$$

### Uncertainty and Errors

A summary of the rules for determining uncertainties and their propagation through a series of operation is given in Table 3-1