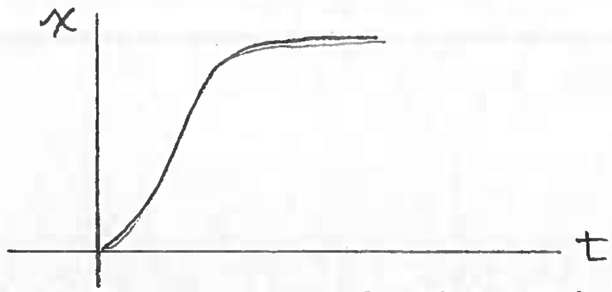
Physics 2100 Laboratory
Lab 2 Calculus Review

A car is moving in one dimension along the x axis. The parameters of motion for the car are:

- $x(t)$ – the displacement of the car along the x axis as a function of time;
- $v(t)$ – the velocity of the car as a function of time; and
- $a(t)$ – the acceleration of the car as a function of time.

The following is a review of how calculus will be used to describe the motion of the car in terms of the parameters of motion- $x(t)$, $v(t)$ and $a(t)$.

Consider that the displacement $x(t)$ for the car is:



The velocity, which is the rate of change of x with time, is the slope of the $x(t)$ versus t curve. We will talk about the average velocity and the instantaneous velocity.

The average velocity at a time t_2 is:

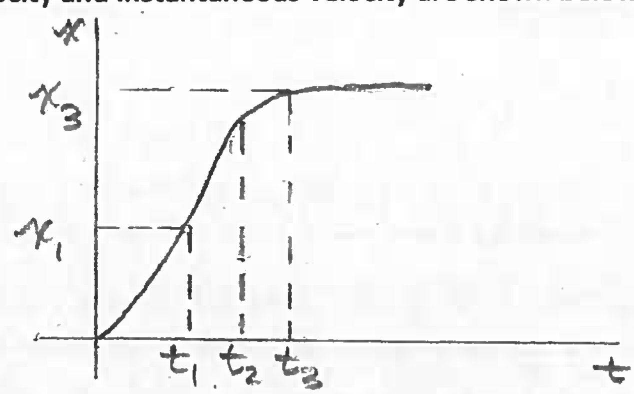
$$v_{avg} = (x_3 - x_1)/(t_3 - t_1) = \Delta x/\Delta t.$$

The instantaneous velocity is:

$$v = \lim_{\Delta t \to 0} \Delta x/\Delta t = dx/dt$$ (the derivative of x as a function of t.)

The instantaneous velocity at a time t_2 is the slope of the $x(t)$ versus t curve at the time t_2.

The average velocity and instantaneous velocity are shown below at time t_2.

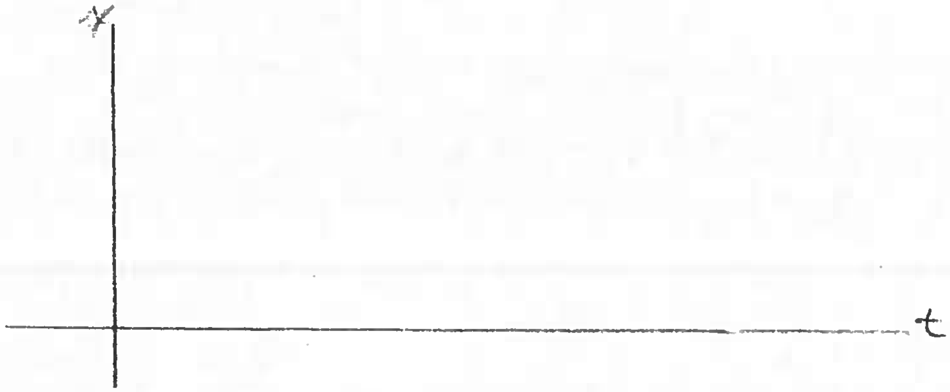


The instantaneous acceleration is the rate of change of $v(t)$ with time.

$$a(t) = dv/dt = d^2x/dt^2$$

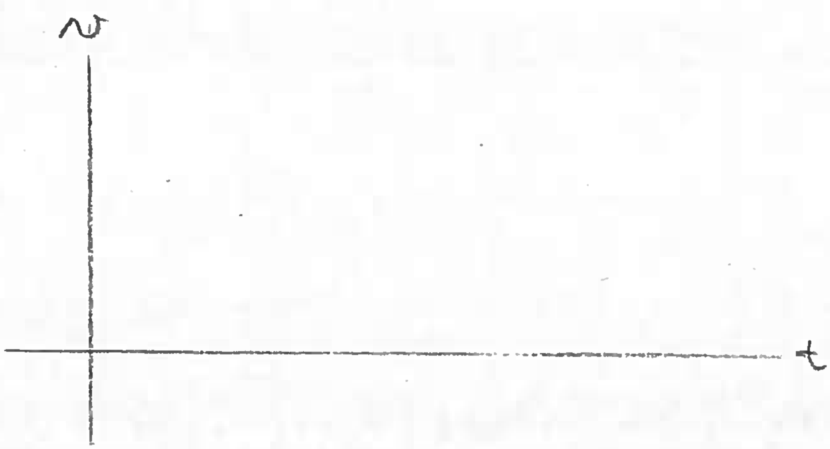
So, given x(t) we can derive v(t) and a(t) by differentiation.

For example, the displacement of a car moving along the x axis is $x(t) = x_0 + At^2$. Assuming A is a positive number, graph x versus t.



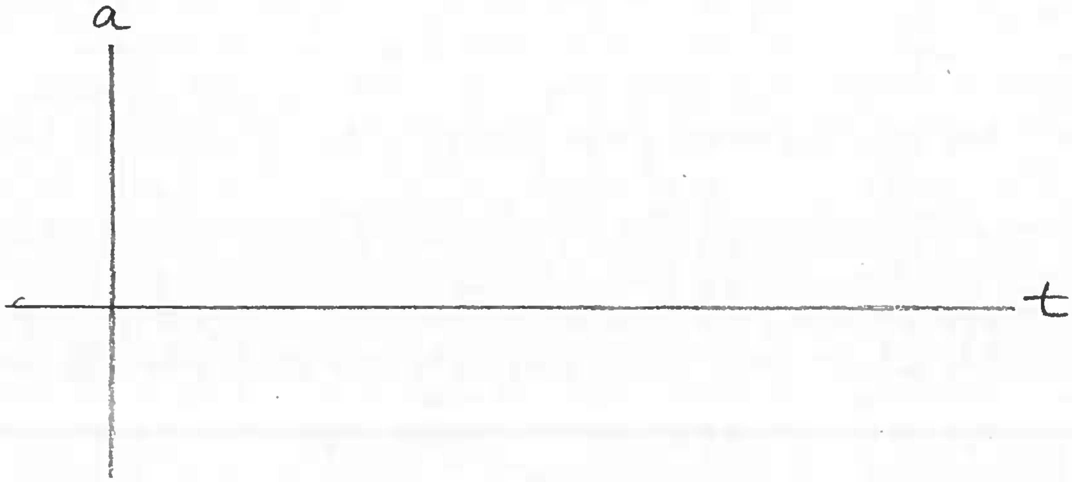
Now find the equation for the velocity of the car, v(t), by differentiation.

Graph v(t) versus t.



Now find the equation for the acceleration, a(t), by differentiation.

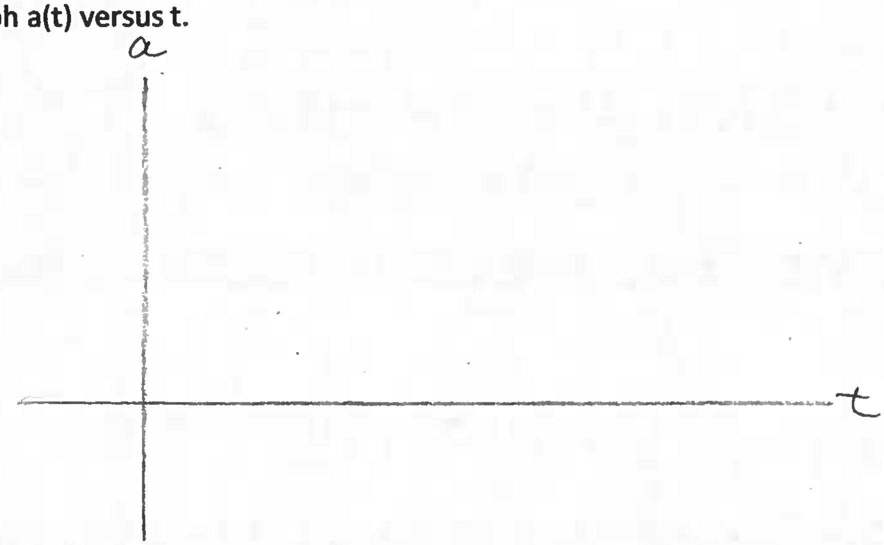
Graph a(t) versus t.



If the acceleration, a(t), is given, integration can be used to derive v(t) and x(t). The following example is a case where the acceleration is a constant, does not change with time. In the 2100 lecture this is the case. In general, acceleration can change with time.

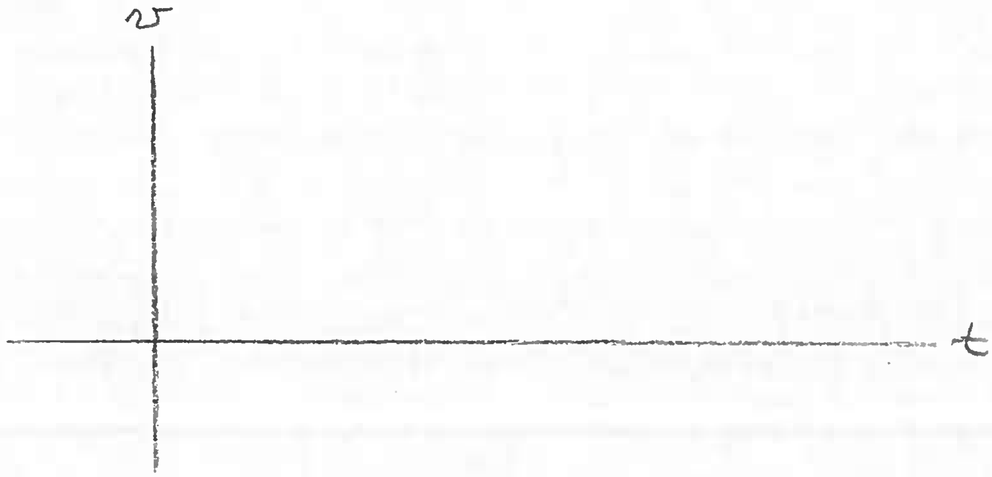
Example: The car travels on the x axis with an acceleration of $a = B$ where B is a positive constant. At t_i=0 seconds, x=0 meters and v=0 meters/s.

Graph a(t) versus t.



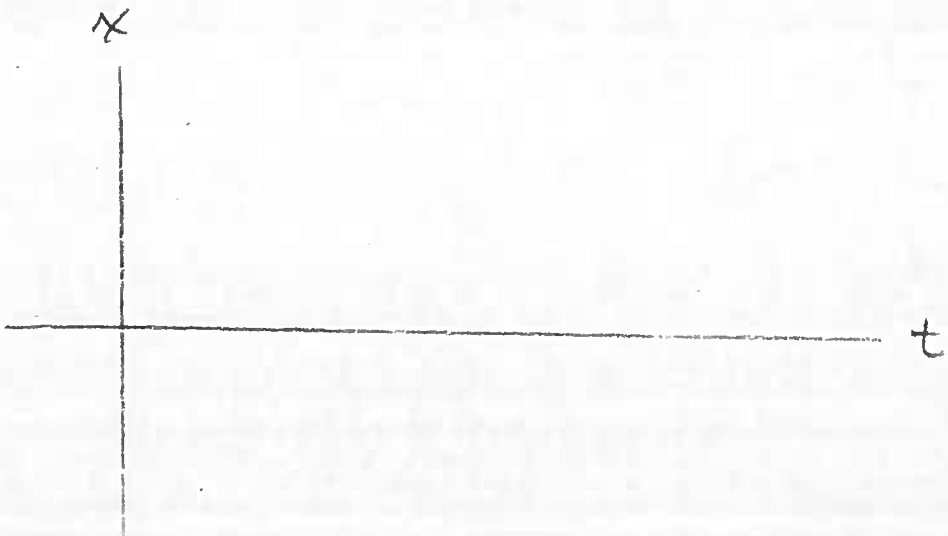
Find the equation for v(t).

Graph v(t) versus t.



Now find the equation for x(t) versus t.

Graph x(t) versus t.

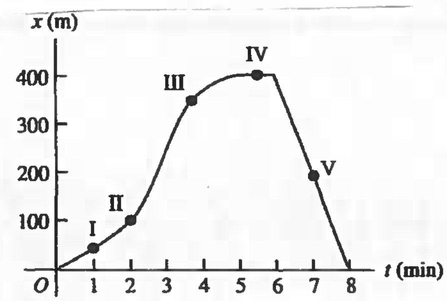


Exercise 2.7 in Edition 15 of the textbook. A car is stopped at a traffic light. It then travels along a straight road such that its distance from the light is given by

$x(t) = bt^2 - ct^3$, where $b = 2.40\ m/s^2$ and $c = 0.120\ m/s^3$.

a. Calculate the average velocity of the car for the time interval t = 0.0 s to t = 10.0 s.
b. Calculate the instantaneous velocity of the car at t = 0.0 s, t = 5.0 s, and t= 10.0 s.
c. How long after starting from rest is the car again at rest?

Exercise 2.10 in Edition 15 of the textbook. A physics professor leaves her house and walks along the sidewalk toward campus. After 5 minutes it starts to rain, and she returns to her house. Her distance from her house as a function of time is shown below.



At which of the labeled points is her velocity

a. zero?
b. constant and positive?
c. constant and negative?
d. increasing in magnitude?
e. decreasing in magnitude?

Exercise 2.52 in Edition 15 of the textbook. The acceleration of a bus is in the x direction and is given by a(t) = At, where $A = 1.2\ m/s^3$.

1. If the bus's velocity at time t = 1.0 s is 5.0 m/s, what is its velocity at time t = 2.0 s?
2. If the bus's position at time t = 1.0 s is 6.0 m, what is its position at time t = 2.0 s?
3. Sketch a(t), v(t) and x(t) for the motion.