California State University – Los Angeles

Department of Mathematics

Master's Degree Comprehensive Examination

Analysis Sample Exam

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Do at least two (2) problems from Section 1 below, and at least three (3) problems from Section 2 below. All problems count equally. If you attempt more than two problems from Section 1, the best two will be used. If you attempt more than three problems from Section 2, the best three will be used. Be sure to show your work for all answers.

(1) Write in a fairly soft pencil (number 2) (or in ink if you wish) so that your work will duplicate well. There should be a supply available.
(2) Write on one side of the paper only.
(3) Begin each problem on a new page.
(4) Assemble the problems you hand in in numerical order.

Exams are graded anonymously, so put your name only where directed and follow any instructions concerning identification code numbers.

SECTION 1 – Do two (2) problems from this section. If you attempt all three, then the best two will be used for your grade.

Sample #1. Let $\mathbb{R}$ denote the set of real numbers, and let $\mathbb{Q}$ denote the set of rational numbers.

Define $f\colon \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

Prove that for all $a \in \mathbb{R}$, we have that f is not continuous at a.

Sample #2. Use the definition of limits to show that

$$\lim_{n\to\infty} \frac{1}{(n+1)^2} = 0.$$

Sample #3. Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences in $\mathbb{R}$. Show that

$$\liminf(x_n + y_n) \geq \liminf x_n + \liminf y_n.$$

SECTION 2 – Do three (3) problems from this section. If you attempt more than three, then the best three will be used for your grade.

Sample #4. Show that the function

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$$

is *not* Riemann integrable on $[0,1]$.

Sample #5. Let μ^* be Lebesgue outer measure on $\mathbb{R}$. Show that

$$\mu^*(\mathbb{Q}) = 0.$$

Sample #6. Consider the measure space $(\mathbb{R}, \mathcal{B}, \mu)$, where $\mathcal{B}$ is the Borel σ-algebra, and μ is Lebesgue measure. For $b, m \in \mathbb{R}$, let $f : \mathbb{R} \to \mathbb{R}$ be the linear function $f(x) = mx + b$. Show that f is measurable.

Sample #7. Let μ be the measure defined on the power set of $\mathbb{R}$ given by

$$\mu(A) = \begin{cases} 1 & \text{if } 0 \in A, \\ 0 & \text{otherwise.} \end{cases}$$

If $\phi : \mathbb{R} \to \mathbb{R}$ is a simple function, show that

$$\int_{\mathbb{R}} \phi \, d\mu = \phi(0).$$