Math 2150 - Test 1 - Fall 2025

Name:____

Directions:

Show steps for full credit.
Also so I can give you partial credit if needed.

Score			
1		2	
3		4	
5		6	
7		Total	

Separable	$f(x) = g(y)\dfrac{dy}{dx}$	Separate $\dfrac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x,y) + N(x,y) \cdot y' = 0$	Test: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ Find f where $\dfrac{\partial f}{\partial x} = M$ and $\dfrac{\partial f}{\partial y} = N$ Solution: $f(x,y) = c$
Constant coefficient	$a_2y'' + a_1y' + a_0y = 0$	Two real roots r_1, r_2 use e^{r_1x} and e^{r_2x} If double real root r use e^{rx} and xe^{rx} If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x}\cos(\beta x)$ and $e^{\alpha x}\sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$ y_1 and y_2 sols. to homogeneous eqn.	$y_p = v_1y_1 + v_2y_2$ $v_1 = \int \dfrac{-y_2 \cdot b(x)}{W(y_1,y_2)}dx$ $\quad v_2 = \int \dfrac{y_1 \cdot b(x)}{W(y_1,y_2)}dx$
Reduction of Order	$y'' + a_1(x)y' + a_0(x)y = 0$ on the interval I	y_1 is a solution that isn't zero on I $y_2 = y_1 \cdot \int \dfrac{e^{-\int a_1(x)dx}}{y_1^2}dx$
Euler's method	$y' = f(x,y)$ $y(x_0) = y_0$	$x_n = x_{n-1} + h$ $y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$

$b(x)$	y_p guess for undetermined coefficients
constant	A
degree one polynomial such as: $5x - 3$ or $2x$	$Ax + B$
degree two polynomial such as: $10x^2 - x + 1$ or $x^2 + x$ or $2x^2 - 3$	$Ax^2 + Bx + C$
$\sin(kx)$ where k is a constant	$A\cos(kx) + B\sin(kx)$
$\cos(kx)$ where k is a constant	$A\cos(kx) + B\sin(kx)$
exponential such as: e^{kx} or $-2e^{kx}$	Ae^{kx}
degree one poly times exponential such as: xe^{kx} or $(2x+1)e^{kx}$	$(Ax + B)e^{kx}$

$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$ quadratic formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

Power series / Taylor series: $f(x_0) + f'(x_0)(x - x_0) + \dfrac{f''(x_0)}{2!}(x - x_0)^2 + \dfrac{f'''(x_0)}{3!}(x - x_0)^3 + \dfrac{f''''(x_0)}{4!}(x - x_0)^4 + \ldots$

$(fg)' = f'g + fg'$ $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$ $\int u\,dv = uv - \int v\,du$ $\frac{d}{dx}\cos(x) = -\sin(x)$ $\frac{d}{dx}\sin(x) = \cos(x)$

$\int \sin(x)dx = -\cos(x)$ $\int \cos(x)dx = \sin(x)$ $\int \dfrac{dx}{1 + x^2} = \tan^{-1}(x)$ $\int \tan(x)dx = \ln|\sec(x)|$

1. [10 points] Consider the following ODE:

$$y''' + \sin(y)y'' + y = \sin(x)$$

(a) What is the order of the equation?

(b) Is it linear or not linear? Wny or why not?

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2. [5 points] Suppose you know that $y_1 = e^{2x}$ and $y_2 = e^{5x}$ are linearly independent solutions to $y'' - 7y' + 10y = 0$ and that $y_p = 6e^x$ is a particular solution to $y'' - 7y' + 10y = 24e^x$. You do not have to verify any of this, just use it to answer the next question below.

State a formula for the general solution to $y'' - 7y' + 10y = 24e^x$.

3. [10 points] Solve the linear equation

$$y' + \frac{1}{x}\, y = 3x^2 - \frac{1}{x}$$

on $I = (0, \infty)$

4. [10 points] Find a solution to the separable initial value problem

$$\frac{dy}{dx} = 3x^2y^2 \qquad y(0) = 1$$

Solve for y in terms of x in your solution.

5. [10 points]

(a) Show that

$$(2xy + 3) + (x^2 + 4y)y' = 0$$

is an exact equation.

(b) Find a solution to the equation above in part (a).

6. [10 points] Find the general solution to

$$y'' + 8y' + 16y = 0$$

7. [10 points] Let $f_1(x) = x$ and $f_2(x) = x^3$. Let $I = (0, \infty)$.

(a) Show that f_1 and f_2 are linearly independent on I.

(b) Show that both f_1 and f_2 solve $x^2y'' - 3xy' + 3y = 0$ on I.

(c) State the general solution to $x^2y'' - 3xy' + 3y = 0$ on I.

Extra page if you need it....