1

(a) $\varphi(3) = \varphi(1) + \varphi(1) + \varphi(1) = 4 + 4 + 4 = 12$

(b) $\varphi(-1) = -\varphi(1) = -4$

---

2

(a) Let $a, b \in \mathbb{Z}$.

Then,

$$\varphi(a+b) = -2(a+b) = (-2a) + (-2b) = \varphi(a) + \varphi(b)$$

(b) $-2n = 0$ iff $n = 0$

Thus, $\ker(\varphi) = \{0\}$

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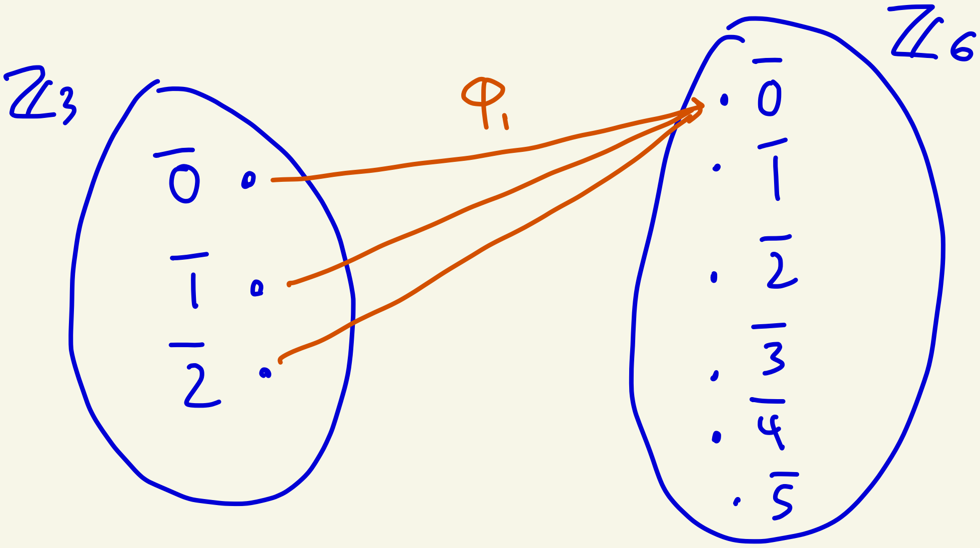
3 $\mathbb{Z}_3$ has generator $\bar{1}$ of order 3.

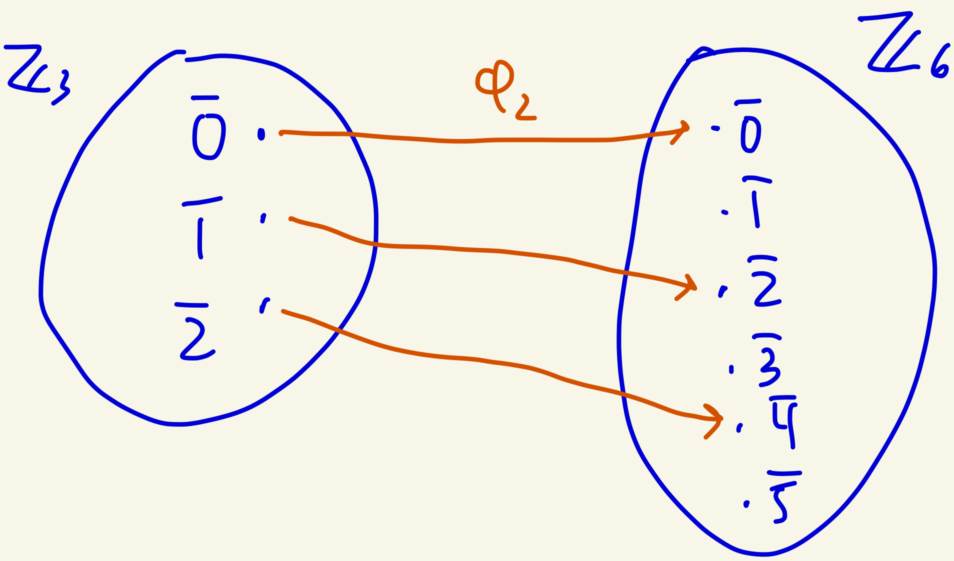
Let's find the elements of $\mathbb{Z}_6$ of order dividing 3.

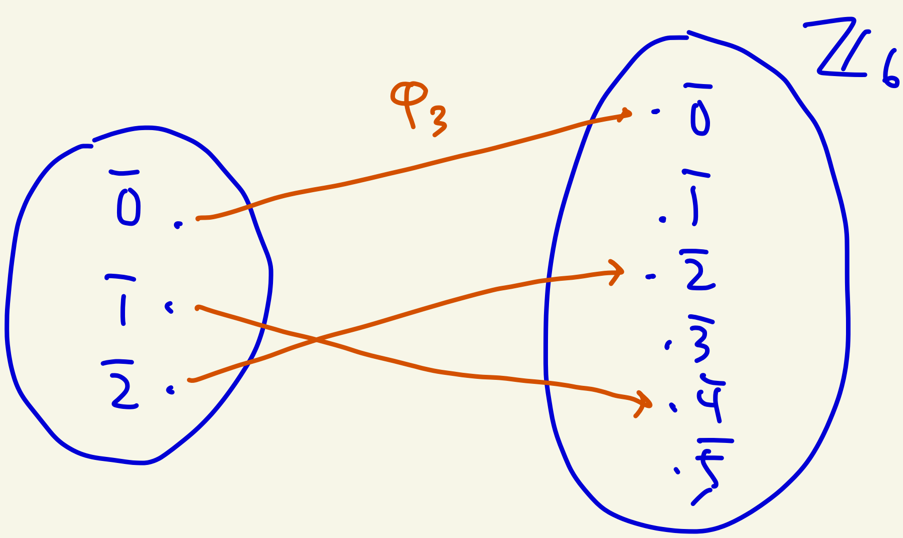
| $\mathbb{Z}_6$ | order |
|---|---|
| $\bar{0}$ | 1 |
| $\bar{1}$ | 6 |
| $\bar{2}$ | 3 |
| $\bar{3}$ | 2 |
| $\bar{4}$ | 3 |
| $\bar{5}$ | 6 |

$\bar{0}, \bar{2}, \bar{4}$ have orders that divide 3.

If $\varphi: \mathbb{Z}_3 \to \mathbb{Z}_6$ is a homomorphism, then $\varphi(\bar{1}) = \bar{0}, \bar{2}$, or $\bar{4}$.




these are the three possibilities

4

(a) $\bar{0}+H=\{\bar{0},\bar{4}\}$ ← identity

$\bar{1}+H=\{\bar{1},\bar{5}\}$

$\bar{2}+H=\{\bar{2},\bar{6}\}$

$\bar{3}+H=\{\bar{3},\bar{7}\}$

4 elements

(b) $\bar{1}+H$ is the inverse of $\bar{3}+H$ since

$$(\bar{1}+H)+(\bar{3}+H)=\bar{4}+H=\bar{0}+H$$

(c) $(\bar{2}+H)+(\bar{2}+H)=\bar{4}+H=\bar{0}+H$

Thus, $\bar{2}+H$ has order 2.

(d) Yes the group is cyclic. $\bar{1}+H$ is a generator

$\bar{1}+H$

$(\bar{1}+H)+(\bar{1}+H)=\bar{2}+H$

$(\bar{1}+H)+(\bar{1}+H)+(\bar{1}+H)=\bar{3}+H$

$(\bar{1}+H)+(\bar{1}+H)+(\bar{1}+H)+(\bar{1}+H)=\bar{4}+H=\bar{0}+H$

(A) HW 4 - #6(c)

(B) HW 6 - #7

(C) HW 6 - #8(d)

(D) HW 4 - #6(e)