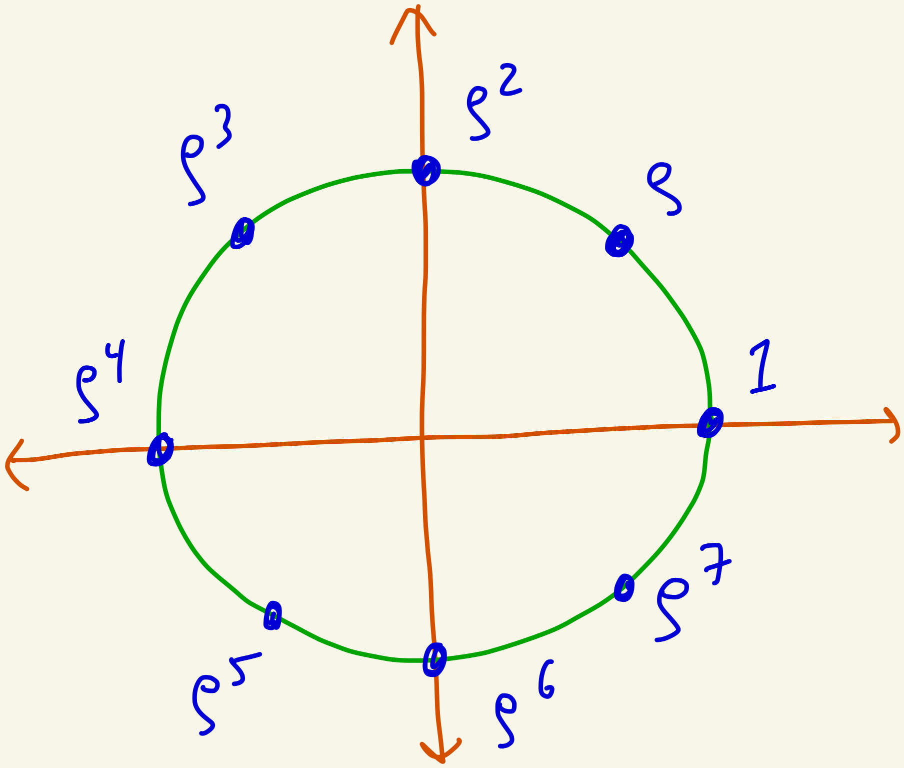
①

(a)



(b) $\rho^3 \cdot \rho^5 = \rho^8 = 1$

Thus, $(\rho^3)^{-1} = \rho^5$

(c) $H = \{1, \rho^2, \rho^4\}$

Not a subgroup since not closed since $\rho^2 \cdot \rho^4 = \rho^6 \notin H$

2

(a) $r^{-3} s \boxed{r^5} s \boxed{r^4} = r^{-3} s r s = s r^3 r s$

$r^4 = 1$

$r^5 = r^4 \cdot r = r$
since $r^4 = 1$

$$\begin{aligned} &= s r^4 s \\ &= s \cdot 1 \cdot s \\ &= s^2 = 1 \end{aligned}$$

(b) $s r^3 \neq 1$

$$\begin{aligned} (s r^3)(s r^3) = s r^3 s r^3 &= s s r^{-3} r^3 \\ &= s^2 r^0 \\ &= 1 \cdot 1 = 1 \end{aligned}$$

Thus, $s r^3$ has order 2

(c)

$\langle r^2 \rangle = \{1, r^2\}$

3 (a)

$$\mathbb{Z}_3 \times \mathbb{Z}_3 = \{(\bar{0},\bar{0}), (\bar{0},\bar{1}), (\bar{0},\bar{2}), (\bar{1},\bar{0}), (\bar{1},\bar{1}), (\bar{1},\bar{2}), (\bar{2},\bar{0}), (\bar{2},\bar{1}), (\bar{2},\bar{2})\}$$

(b)

$$(\bar{1},\bar{2}) \neq (\bar{0},\bar{0})$$

$$(\bar{1},\bar{2}) + (\bar{1},\bar{2}) = (\bar{2},\bar{4}) = (\bar{2},\bar{1}) \neq (\bar{0},\bar{0})$$

$$(\bar{1},\bar{2}) + (\bar{1},\bar{2}) + (\bar{1},\bar{2}) = (\bar{3},\bar{6}) = (\bar{0},\bar{0})$$

Thus, $(\bar{1},\bar{2})$ has order 3

(c)

$$(\bar{1},\bar{1}) + (\bar{2},\bar{2}) = (\bar{3},\bar{3}) = (\bar{0},\bar{0})$$

Thus, $(\bar{2},\bar{2})$ is the inverse of $(\bar{1},\bar{1})$

(A or B)

A See HW 2 #12

B See HW 1 #11

(c) This operation has no identity element.

Can we find $e \in \mathbb{R}^*$ where $e * a = a$ for all $a \in \mathbb{R}^*$?

This would need $|ea| = a$ for all $a \in \mathbb{R}^*$.

But if $a = -1$ for example we would have $|-e| = -1$.

This can't happen since absolute value is never negative.

Thus, there is no identity element and $\mathbb{R}^*$ is not a group under this operation.

(D) HW 3 - #6
