1 (a) order 3

(b) not linear because of the $\sin(y)\, y'$ term.

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2 (HW 6 - Problem 1(d))

$$y = y_h + y_p = c_1 e^{2x} + c_2 e^{5x} + 6e^x$$

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(3) $\boxed{\text{HW } 3\text{-}1(d)}$

$$y' + \frac{1}{x}y = 3x^2 - \frac{1}{x}$$

$$I = (0, \infty) \leftarrow \boxed{0 < x}$$

$$A(x) = \int \frac{1}{x}\,dx = \ln|x| = \ln(x)$$

$\boxed{x > 0}$

multiply ODE by $e^{A(x)} = e^{\ln(x)} = x$ to get:

$$xy' + y = 3x^3 - 1$$

$$(xy)' = 3x^3 - 1$$

$$xy = \int (3x^3 - 1)\,dx$$

$$xy = \frac{3}{4}x^4 - x + C$$

$$y = \frac{3}{4}x^3 - 1 + \frac{C}{x}$$ where C can be any constant

4 $\dfrac{dy}{dx} = 3x^2y^2$

$$\frac{dy}{y^2} = 3x^2dx$$

$$\int y^{-2}\,dy = \int 3x^2dx$$

$$-y^{-1} = x^3 + C$$

$$-\frac{1}{y} = x^3 + C$$

$y(0)=1$ implies:

$$-\frac{1}{1} = 0^3 + C$$

$$-1 = C$$

So, $-\dfrac{1}{y} = x^3 - 1$

Thus, $y = \dfrac{-1}{x^3-1}$

5 (a)

$$M = 2xy + 3 \qquad \frac{\partial M}{\partial x} = 2y \qquad \frac{\partial N}{\partial x} = 2x$$

$$N = x^2 + 4y \qquad \frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial y} = 4$$

all continuous on all of the xy-plane

Since $\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$ the equation is exact

(b)

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= 2xy + 3 \qquad (*) \\ \frac{\partial f}{\partial y} &= x^2 + 4y \qquad (**) \end{aligned}}$$

(*) gives $f(x,y) = x^2y + 3x + c(y)$

(**) gives $f(x,y) = x^2y + 2y^2 + D(x)$

Thus, $x^2y + 3x + c(y) = x^2y + 2y^2 + D(x)$

So, $3x + c(y) = 2y^2 + D(x)$

Thus, $f(x,y) = x^2y + 3x + 2y^2$.

$\boxed{\text{Answer: } x^2y + 3x + 2y^2 = c \text{ where } c \text{ is any constant}}$

6 $\boxed{\text{HW 7 \#1(e)}}$

$$y'' + 8y' + 16y = 0$$

$$r^2 + 8r + 16 = 0$$

$$r = \frac{-8 \pm \sqrt{8^2 - 4(1)(16)}}{2(1)} = \frac{-8 \pm \sqrt{0}}{2} = -4$$

$$y_h = c_1 e^{-4x} + c_2 x e^{-4x}$$

⑥ (a)

$$W(f_1,f_2) = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = x(3x^2) - (x^3)(1) = 2x^3$$

This is not the zero function on $I = (0,\infty)$

For example when $x=1$ we have $2x^3 = 2(1)^3 = 2 \neq 0$.

Thus, f_1 and f_2 are linearly independent on I.

(b) $f_1 = x$, $f_1' = 1$, $f_1'' = 0$

Thus, $x^2y'' - 3xy' + 3y = x^2(0) - 3x(1) + 3x = 0$

So, $f_1(x) = x$ solves the equation.

$f_2 = x^3$, $f_2' = 3x^2$, $f_2'' = 6x$

Thus, $x^2 f_2'' - 3x f_2' + 3f_2 = x^2(6x) - 3x(3x^2) + 3x^3$

$$= 6x^3 - 9x^3 + 3x^3 = 0$$

So, f_2 solves the equation.

(c) $y_h = c_1 x + c_2 x^3$

where c_1, c_2 are any constants
