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How to Be an Anti-Platonist

Abstract: This paper develops and defends an anti-platonist view of mathematics —in particular, an error-theoretic view of mathematics. In addition, it is argued that the error theory developed here is superior to Wittgenstein's philosophy of mathematics because it avoids a mistake that Wittgenstein made about the interpretation of mathematical discourse, while at the same time preserving some of Wittgenstein's insights about mathematics. Finally, it is also argued here that a certain sort of mathematical *relativism* is true, regardless of whether platonism is true, and that, perhaps surprisingly, the best versions of platonism entail mathematical relativism.

Keywords: Platonism, mathematics, abstract objects, Wittgenstein

1 Introduction

In this paper, I will develop and defend an anti-platonist view of mathematics—in particular, an error-theoretic view—and I will argue that it is superior to Wittgenstein's philosophy of mathematics. In §2, I will present what I think is the strongest argument for platonism. Then in §§3–5, I will explain how we can sidestep this argument and plausibly endorse an anti-platonist view, in particular, an error-theoretic view. In §6, I will argue that my view enables us to save various Wittgensteinian insights about mathematics while avoiding a mistake that I think he made about mathematics. Finally, in §7, I will draw out three consequences of my argument, namely, (a) that my error-theoretic view is superior to Wittgenstein's philosophy of mathematics; and (b) that a certain sort of *mathematical relativism* is true, regardless of whether platonism is true; and (c) that, somewhat surprisingly, the best versions of platonism actually entail mathematical relativism.

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2 Preliminaries

2.1 The Argument for Platonism

Platonism is the view that abstract objects exist—where an *abstract object* is a non-physical, non-mental, unextended, acausal, non-spatiotemporal object. The best argument for platonism is based on the following thesis:

> *The platonistic interpretation of mathematical discourse:* Our mathematical sentences and theories are about (or at least *purport* to be about) abstract objects. For instance, the sentence '3 is prime' makes a claim (or at least purports to make a claim) about a specific abstract object, namely, the number 3.

This thesis does not entail platonism all by itself—i.e., it does not entail that abstract objects actually *exist*[1]—but the platonistic interpretation of mathematics is the central premise in what is widely thought of as the best argument for platonism. We can formulate this argument as follows:

> [1] The platonistic interpretation of mathematical discourse is true. And
> [2] The sentences of our mathematical theories are true; e.g., '3 is prime' is true. But
> [3] If [1] and [2] are true, then platonism is true. For example, if '3 is prime' makes a claim about a specific abstract object (namely, the number 3), and if this claim is *true*, then the abstract object in question (i.e., the number 3) exists. Therefore,
> [4] Platonism is true.[2]

You might think that anti-platonists should respond to this argument by rejecting the platonistic interpretation of mathematics—i.e., by claiming that our mathematical sentences and theories do *not* make claims about abstract objects. That is how Wittgenstein would respond to this argument. But I do not think that is the best way for anti-platonists to respond. Indeed, I think the platonistic interpretation is not just true but *obviously* true. I will not attempt to argue for this claim in full here, but I would like to say a few words about it.

1 Here is an analogy: the *theistic interpretation of ordinary Christian 'God' discourse*—which says that the ordinary Christian word 'God' refers (or at least *purports* to refer) to an all-powerful creator of the universe—does not entail theism (i.e., it does not entail that God actually exists).
2 Arguments of this kind have been endorsed by numerous philosophers, most notably Frege (1884, 1893–1903).

2.2 The Argument for the Platonistic Interpretation of Mathematics

The main argument for the platonistic interpretation of mathematics is based on the claim that (i) our mathematical sentences should be read at *face value*—so, e. g., '3 is prime' should be read as being of the form '*Fa*' and, hence, as making a claim (or at least as purporting to make a claim) about a certain specific object, namely, the number 3—and (ii) these sentences do not make claims (or even purport to make claims) about physical or mental objects, so that if they make claims about objects at all, then they make claims about *abstract* objects.[3] The argument for this two-pronged claim proceeds by showing how implausible the alternative views are. The only alternatives are (a) the *physicalistic* view that sentences like '3 is prime' make claims about specific physical objects; (b) the *psychologistic* view that these sentences make claims about specific mental objects in specific human heads, e. g., Madonna's idea of the number 3; and (c) the *non-literalist* view that these sentences should not be read at face value and that, on the correct reading of these sentences, they do not make claims about specific objects at all (so, e. g., on this view, '3 is prime' is not of the form '*Fa*', and it does not make a claim about the number 3).[4,5]

I think there are good arguments against all three of these views. The problem with them, in a nutshell, is that they all involve empirical hypotheses about the meanings of ordinary mathematical utterances, in ordinary English, that are ex-

3 I am assuming that all objects are physical, mental, or abstract. I suppose you might think there are also *social* objects; but they presumably decompose into physical, mental, and abstract objects. Consider, e. g., the convention to stop at red lights; if this is not an abstract object, then it presumably reduces to individual mental things—i. e., to beliefs and intentions and so on of actual people.

4 Mill (1843) and Kitcher (1984) endorsed views with physicalistic leanings, and Brouwer (1912, 1948) and Heyting (1956) endorsed views with psychologistic leanings; but all of their views had non-literalist threads running through them as well. And since '3' clearly does not denote a specific physical object or a specific mental object in a specific human head, it seems that advocates of physicalistic and psychologistic views more or less *have to* endorse some sort of non-literalism.

5 There are two kinds of non-literalism: *paraphrase* views say that mathematical sentences make non-face-value claims, and *non-cognitivist* views say that they do not make genuine claims at all. Paraphrase views have been endorsed by Putnam (1967a, 1967b), Horgan (1984), Hellman (1989), Chihara (1990), Yi (2002), Hofweber (2005), Rayo (2008, 2013), Dorr (2008), and Moltmann (2013). One might also interpret the early Hilbert (see, e. g., his (1899) and his letters to Frege in Frege (1980)) and Curry (1951) in this way, and one might think that a paraphrase view can be found in the early Wittgenstein (1921). Non-cognitivist views were endorsed by the 19th-Century game formalists that Frege criticized in his (1884)—e. g., Thomae. I think that Wittgenstein—in both the early period (TLP 1921) and the later period (RFM 1956)—is best interpreted as endorsing non-cognitivism; but Wittgenstein's views were subtle and complex, and they are open to multiple interpretations.

tremely implausible and wildly out of touch with the usage and intentions of ordinary speakers of mathematical language. Consider, e. g., the non-literalist view known as *if-thenism*, which says that, e. g., '3 is prime' says that *if there were numbers, then 3 would be prime.*[6] This is extremely hard to believe. It is just completely implausible to suppose that when ordinary people (i. e., non-philosophers) utter the sentence '3 is prime', what they are really saying is that if there were numbers, then 3 would be prime. This just seems to get wrong what ordinary people actually mean when they utter sentences like this.

(If-thenists might respond by claiming that their view concerns not what ordinary people actually mean by their mathematical utterances, but what they *should* mean.[7] But if if-thenists say this, then their view is totally irrelevant in the present context because the thesis that I am talking about here (namely, the platonistic interpretation of mathematics) is—I hereby stipulate—a claim about the actual meanings of mathematical sentences in ordinary mathematical discourse. So in order for if-thenism to be relevant here, it has to be about the ordinary-language meanings of mathematical sentences. But, again, if we read if-thenism in this way, then it is just completely implausible.)

I think there are good arguments of this kind against *all* of the alternatives to the platonistic interpretation of mathematics. In other words, I think that all of the alternative views are incompatible with the empirical facts about what ordinary people actually mean when they utter mathematical sentences. For example, physicalism and psychologism are both wildly out of touch with the way that ordinary mathematicians talk about infinities. I will not run through the arguments against the various alternatives to the platonistic interpretation here, but they have been articulated numerous times in the past; see, e. g., Frege (1884, 1893–1903), Resnik (1980), and my (1998, 2014).

(You might think that the platonistic interpretation of mathematics is *also* incompatible with what ordinary people mean when they utter sentences like '3 is prime'; for you might think that when ordinary people utter sentences like this, they do not intend to be talking about abstract objects. But advocates of the platonistic interpretation do not have to say that ordinary people have positive intentions to be talking about abstract objects. Instead, they can (and should) say that (i) there are features of the intentions of typical mathematicians (and ordinary folk) that are inconsistent with all non-platonistic interpretations of mathematics, and (ii)

6 Views like this have been endorsed by Putnam (1967a,b), Horgan (1984), Hellman (1989), and Dorr (2008).

7 One non-literalist who denies that he is trying to capture the actual meanings of real mathematical utterances is Chihara (2004).

there is nothing in the intentions of typical mathematicians (or ordinary folk) that is inconsistent with the platonistic interpretation.)

2.3 Mathematical Error Theory

So I do not think anti-platonist's should reject the platonistic interpretation of mathematical discourse. Instead, I think they should reject premise [2] in the above argument against platonism, and I think they should endorse the following view:

> *Mathematical error theory:* (a) Our mathematical sentences and theories do purport to be about abstract objects, but (b) there are no such things as abstract objects, and so (c) our mathematical sentences and theories are false (or at least not true[8]). (Thus, on this view, just as *Alice in Wonderland* is not literally true because (among other reasons) there are no such things as talking rabbits, hookah-smoking caterpillars, and so on, so too our mathematical theories are not literally true because there are no such things as numbers and sets and so on.)[9]

You might think that this view is completely implausible because we have good reason to think that our mathematical theories are true. But I am going to argue in what follows that, despite appearances to the contrary, we do not have any good reason to think that our mathematical theories are strictly speaking true.

Now, I suppose you might think it is just *obvious* that our mathematical theories are true, but in fact, it is not. Given the platonistic interpretation of mathematics, it follows that our mathematical theories are true only if there actually exist abstract objects; thus, since it is not obvious that there are abstract objects, it is also not obvious that our mathematical theories are true. Thus, if you want to claim that our mathematical theories are true, you need an *argument.*[10]

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8 One might think, with Strawson (1950), that if there are no mathematical objects (e. g., if there is no such thing as the number 3), then mathematical sentences like '3 is prime' are *neither true nor false*—because they have a false presupposition. I prefer the view that, in this scenario, '3 is prime' is *false*, and I will talk that way in this paper, but nothing important turns on this.

9 Field introduced this view in his (1980).

10 You might claim that sentences like '3 is prime' have *Moorean* status. But error theorists can claim that (i) "'3 is prime' is a claim about the number 3 and it could be true only if the number 3 actually existed" and (ii) 'There is no such thing as platonic heaven' also have Moorean status. The problem is that there are three obvious-seeming sentences here that are jointly inconsistent; so the challenge is to figure out which of them is actually false. Now, of course, platonists can demand that error theorists account for the *seeming obviousness* of sentences like '3 is prime.' But error theorists can presumably do this by pointing out that (a) the only way in which these sentences

There are two main arguments for the truth of our mathematical theories—*the Quine-Putnam indispensability argument*, and what I will call *the objective-correctness argument.* I will respond to both of these arguments here, and in the process, I will argue that error theory is much more plausible than it seems at first blush. Indeed, I am going to argue that error theorists can endorse a view of the objectivity, correctness, and applicability of mathematics that is *independently plausible*—i. e., plausible independently of any ontological worries about the existence of abstract objects.

I will respond to the objective-correctness argument in §§3 and 4 and the Quine-Putnam argument in §5. I will spend more time discussing the objective-correctness argument because my discussion of that argument will enable me to give a relatively quick response to the Quine-Putnam argument.

3 A FAPP-ist Account of Objective Mathematical Correctness

3.1 The Objective-Correctness Argument against Error Theory

The first argument for the truth of our mathematical theories—and, hence, against error theory—can be formulated as follows:

> *The Objective-Correctness Argument:* We need to endorse the truth of our mathematical theories in order to account for the obvious fact that mathematics is an objective, factual discipline. There is an obvious sense in which mathematical sentences like '3 is prime' are *right*, or *correct*, or some such thing, whereas sentences like '4 is prime' are *wrong*, or *incorrect.* Moreover, it does not seem that we are just making this up; it seems to be an *objective fact* that '3 is prime' is right and '4 is prime' is wrong. And it seems that the only way to account for this is to say that '3 is prime' is *true* and '4 is prime' is *false.*

3.2 FAPP-Truth

I think that error theorists can respond to the objective-correctness argument by arguing that (a) while sentences like '3 is prime' are *strictly speaking* false, there is nevertheless an objective sort of *correctness* that attaches to these sentences

could be false is if numbers just do not exist, and (b) most people do not ever think about this possibility, and so (c) it can easily seem to us that these sentences are necessarily true.

—or a sort of *for-all-practical-purposes truth*—and (b) we can use the fact that our mathematical sentences and theories are for-all-practical-purposes true in this way to account for the objectivity and factualness of mathematics and for the difference between sentences like '3 is prime' and sentences like '4 is prime'.

The first thing that error theorists need to do here is to define the relevant sort of correctness, or for-all-practical-purposes truth. Roughly speaking, I think error theorists should say that a sentence is for-all-practical-purposes true—or, for short, *FAPP-true*—if and only if it would have been true if platonism had been true, i.e., if abstract objects like numbers and sets and so on had existed. To make this more precise, error theorists need to say more about the kind of platonism that they are appealing to here. I think there are good reasons for error theorists to define FAPP-truth in terms of the following version of platonism:

> *Plenitudinous platonism:* There actually exists a plenitude of abstract objects; i.e., there actually exist abstract objects of all possible kinds.

I think there are good reasons—reasons that I have articulated elsewhere[11]—for thinking that plenitudinous platonism is the best version of platonism, and because of this, I think error theorists should define FAPP-truth in terms of plenitudinous platonism, rather than some non-plenitudinous version of platonism. In particular, I think they should define it as follows:

> A sentence is *for-all-practical-purposes true*—or, for short, *FAPP-true*—if and only if it would have been true if there had actually existed a plenitudinous realm of abstract objects. (Or alternatively: a sentence is FAPP-true if and only if it would have been true if plenitudinous platonism had been true.)

If you like, you can think of FAPP-truth as being equivalent to *truth in the story of abstract objects*—where "the story of abstract objects" is just plenitudinous platonism. I am okay with this way of talking, but it is important to note that this is just a metaphor and not the official definition of FAPP-truth. Most importantly, it should be noted that error theorists are not committed to the existence of *stories.*[12]

11 Most notably, I argued in my (1995, 1998, and 2016) that platonists can provide an adequate response to Benacerraf's (1973) challenge—i.e., they can provide an adequate explanation of how we humans could acquire knowledge of abstract objects, despite the fact that we exist wholly within space and time and have no information-gathering contact with those objects—if and only if they endorse plenitudinous platonism.

12 Field (1980, 1989, 1998) defines a notion of *truth in the story of mathematics* that is similar to the notion of FAPP-truth that I am defining here. But it is also importantly different. Somewhat roughly, on Field's view, a sentence is *true in the story of mathematics* iff it follows from currently ac-

3.3 FAPP-ism

Given the above definition of FAPP-truth, error theorists can respond to the objective-correctness argument against their view by arguing for the following theory:

> *FAPP-ism:* (i) Regardless of whether our mathematical theories are strictly speaking true, they are FAPP-true; e. g., sentences like '3 is prime' are FAPP-true. Moreover, (ii) FAPP-truth is a legitimate kind of objective correctness, worthy of the term 'for-all-practical-purposes truth'. Finally, (iii) the fact that our mathematical theories are FAPP-true can be used to account for the objectivity and factualness of mathematics and for the difference between sentences like '3 is prime' on the one hand and sentences like '4 is prime' on the other.

I will use the term *'FAPP-ist error theory'* to denote the conjunction of FAPP-ism and error theory. It is important to note, however, that FAPP-ism is entirely independent of error theory; FAPP-ism could be true even if error theory is false—indeed, even if platonism is true. But the more important point here is that FAPP-ism is compatible with error theory, and if error theorists can argue that FAPP-ism is true—and, as we will see in §3.6, I think they *can*—then they will have a response to the objective-correctness argument against their view.

3.4 Proto-Mathematical Truths

It is important to notice that according to FAPP-ist error theorists, there are counterfactual truths lurking right behind our mathematical theories. For according to this view, to say that, e. g., '3 is prime' is FAPP-true is equivalent to saying that the following counterfactual is *strictly and literally true:*

> [CF] If there had actually existed a plenitudinous realm of abstract objects, then it would have been the case that 3 was prime.

According FAPP-ist error theorists, counterfactuals like [CF] are the truths that are, so to speak, *behind* our mathematical theories. Given this, we can think of these sentences as *proto-mathematical truths.*

Now, you might object to this stance by claiming that (a) the reliance on counterfactuals is going to commit FAPP-ist error theorists to the existence of possible

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cepted mathematical axioms. I think Field's view is problematic in a few ways; e. g., as I argued in my (2009), his view is incompatible with the fact that there can be mathematical sentences that are true (or FAPP-true, or whatever) but undecidable in our current theories.

worlds, and (b) possible worlds are abstract objects. But I will respond to this worry in §4.2.

3.5 Distinguishing FAPP-ist Error Theory from Two Other Views

It is important to distinguish FAPP-ist error theory from the *if-thenist* view that [CF] captures the *real content* of '3 is prime'—i. e., that '3 is prime' *says* that if there had been a plenitudinous mathematical realm, then 3 would have been prime. This is a controversial (and I think implausible) empirical hypothesis about the meaning of '3 is prime' in ordinary English, and I want to emphasize that this is *not* what FAPP-ist error theory says. FAPP-ist error theory does not say that [CF] provides the real content of '3 is prime'; rather, it says that the FAPP-truth of '3 is prime' consists in the literal truth of [CF]. And this is perfectly compatible with the platonistic interpretation of mathematics, which again, says that '3 is prime' makes a claim about an abstract object and that it could be true only if platonism is true.

It is also important to distinguish FAPP-ist error theory from the following view:

> *Revolutionism:* We should change what we are doing in mathematics—e. g., we should start meaning what if-thenists think we mean by sentences like '3 is prime', or some such thing —because our current mathematical theories are not true.

FAPP-ist error theorists of the kind I have in mind do *not* endorse this view. Instead, they endorse the following view:

> *Anti-Revolutionism:* We do not need to change what we are doing in mathematics because *there is nothing wrong with it.* Our mathematical theories are not true, but this does not matter because the mark of goodness in mathematics is not truth; it is FAPP-truth. Thus, since our mathematical theories are FAPP-true, they are good, and so we do not need to change what we are doing.

I will not provide a direct argument in this paper for the claim that anti-revolutionism is superior to revolutionism, but this claim more or less follows from the argument for FAPP-ism that I give in §3.6.[13]

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13 Anti-revolutionism gives error theorists a response to Lewis's claim that it would be laughably presumptuous to suggest that we should reject our mathematical theories for philosophical reasons, given the track records of these two disciplines. Error theorists can respond to this by point-

3.6 The Argument for FAPP-ism

I will argue in this subsection that FAPP-ism is true and that this gives error theorists a response to the objective-correctness argument against their view. FAPP-ism, recall, has three parts—(i), (ii), and (iii)—so I need to argue for all three of these parts.

Part (i) of FAPP-ism says that even if there are no such things as mathematical objects, so that our mathematical theories are not strictly speaking true, these theories are still FAPP-true. To argue for this, I need to argue that (a) mathematical sentences like '3 is prime' are FAPP-true, and (b) this does not require the existence of mathematical objects. But to say that sentences like '3 is prime' are FAPP-true is equivalent to saying that counterfactuals like [CF] are true. Thus, what I need to argue here is that (a) counterfactuals like [CF] are true, and (b) this does not require the existence of mathematical objects. Here is an argument for the truth of counterfactuals like [CF] that does not rely on the claim that mathematical objects exist:

> [CF] is true—indeed, it is *analytic*—because its antecedent analytically entails its consequent. The antecedent of [CF] says that plenitudinous platonism is true. That view entails that there are abstract objects of all possible kinds, and so it entails that there are abstract objects of the kinds that *our mathematical theories* are about (I am assuming here that our mathematical theories are consistent and, hence, that there *could* be objects of the kinds that they are about). Thus, plenitudinous platonism entails that the *natural numbers* exist. But the claim that the natural numbers exist analytically entails the claim that 3 is prime. But this is just the consequent of [CF]. So, again, [CF] is true because its antecedent analytically entails its consequent. And exactly analogous remarks can be made about other counterfactuals like [CF]—e. g., 'If there had actually existed a plenitudinous realm of abstract objects, then it would have been the case that there were infinitely many primes.'

This argument suggests that counterfactuals like [CF] are true, and it does not rely on the claim that abstract objects exist, and so we seem to have what we need—namely, an argument for the claim that (a) these counterfactuals are true, and (b) the truth of these counterfactuals does not require the existence of mathematical objects.

You might object here by saying this: "Abstract objects are going to end up coming in the back door because counterfactuals make claims about *possible worlds*, and possible worlds are abstract objects." I will respond to this worry in §4.2, but for now the point is that abstract objects are not coming in the *front*

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ing out that they are not *criticizing* our mathematical theories. Rather, they are criticizing the *philosophical* view that the mark of goodness in mathematics is truth.

door. In particular, the truth of [CF] does not require the existence of the number 3. Indeed, this was the error theorist's whole reason for switching from sentences like '3 is prime' to counterfactuals like [CF]. Counterfactuals like [CF] make claims not about what mathematical objects *are* like, but about what they *would be like, if they existed.* And so they can be true even if mathematical objects do not exist. And so it seems to me that part (i) of FAPP-ism is true.

Part (ii) of FAPP-ism says that FAPP-truth is a legitimate kind of objective correctness, worthy of the term 'for-all-practical-purposes truth'. The argument for this has really already been given. For to say that sentences like '3 is prime' are FAPP-true is just to say that counterfactuals like [CF] are *true*, and I have already argued for this; indeed, it follows from what I have argued here that counterfactuals like [CF] are *objectively* true—because I have argued that the antecedents of these counterfactuals straightforwardly *entail* their consequents.

But there is also a second point that is worth bringing out here. The mathematical sentences that come out FAPP-true on the error theoretic view—and, indeed, that *are* FAPP-true—are precisely the sentences that come out true on the (plenitudinous) platonist view. In other words, (plenitudinous) platonists and FAPP-ist error theorists divide the mathematical sentences into *good* ones and *bad* ones in an extensionally equivalent way. For instance, '3 is prime' is FAPP-true, and '4 is prime' is not. And so on. (In case you are wondering why '4 is prime' is not FAPP-true, the reason is that even if the natural numbers existed, it would not be the case that 4 was prime.)

So (a) it is an objective fact which mathematical sentences are FAPP-true, and (b) FAPP-truth applies to the exact set of sentences that platonists (and, indeed, just about all of us) think are true. I think that this is enough to give us the result that FAPP-truth is a legitimate kind of *objective correctness*, worthy of the term 'for-all-practical-purposes truth'. And so I think that part (ii) of FAPP-ism is true.

Finally, if parts (i) and (ii) are both true—i. e., if our mathematical sentences and theories are FAPP-true, and if this is a legitimate kind of objective correctness—then this gives FAPP-ist error theorists a way to account for (a) the fact that mathematics is an objective, factual discipline, and (b) the difference between sentences like '3 is prime' and sentences like '4 is prime'. But that is just what part (iii) of FAPP-ism says. So if I am right that parts (i) and (ii) of FAPP-ism are true, then part (iii) is true as well. And since FAPP-ism contains only three parts, it follows from this that FAPP-ism is true. Moreover, if FAPP-ism is true, then it clearly gives error theorists a response to the objective-correctness argument against their view—because it gives them a way to account for the objective correctness of mathematics and for the difference between sentences like '3 is prime' and sentences like '4 is prime'.

This gives me an argument for what I wanted to establish in this subsection. But I want to drive all of this home by arguing that the FAPP-ist view that I have developed here is independently plausible as a view of the objective correctness of our mathematical theories. I will do this by arguing that platonists need to endorse a virtually identical view. This might be surprising. For surely platonists should say that the reason our mathematical theories are "correct" is that they are *true*, and the reason mathematics is an objective, factual discipline is that our mathematical theories provide accurate representations of objective mathematical facts—in particular, facts about abstract objects. But while platonists should indeed say this, I think they should also say something like the following:

> We (platonists) think that counterfactuals like [CF] are true. So we think that our mathematical theories are FAPP-true. Now, of course, we *also* think that these theories are *true*—because we think that mathematical objects like 3 actually *exist*. It is important to note, however, that this existence fact—the fact that mathematical objects actually exist—does not do any work in determining which mathematical sentences are the *correct* ones, or the *good* ones. For this is already completely determined by the proto-mathematical facts that FAPP-ist error theorists believe in—i. e., the facts about which sentences are FAPP-true. Thus, perhaps surprisingly, the existence fact that platonists endorse is not needed in order for there to be an objective sort of correctness that attaches to our mathematical theories, and from a mathematical point of view (as opposed to a metaphysical point of view), the existence fact is completely uninteresting and unimportant.

Why should platonists endorse this view? Well, suppose that, as of right now, there are no mathematical objects (and that there never have been) but that tonight at midnight, God is going to bring a plenitudinous realm of abstract mathematical objects into existence. On this supposition, it follows that, as of right now, sentences like '3 is prime' are not true but that tomorrow they will be. Moreover, it is already determined *which* mathematical sentences are going to become true. This is determined by the proto-mathematical facts that already obtain today. To put the point differently, as of right now (without there being any mathematical objects), we can say that (a) the mathematical sentences are already separated into the good ones and the bad ones, and (b) when the mathematical realm pops into existence at midnight, there will be *no change* in the membership of these two sets, i. e., the good set and the bad set. All that will happen is that the good sentences will go from being FAPP-true to being true (and note that even after they become true, they will still be FAPP-true as well).

So it seems that even if platonists are right that there exists a plenitude of abstract objects, that existence fact does not do anything to determine *which* mathematical sentences are the good ones, or the correct ones. Now, of course, if abstract objects exist, then they make it the case that the good/correct sentences are *true*, instead of just FAPP-true; but the existence of abstract objects does not do anything to determine which mathematical sentences are the *good* ones.

You might respond here by saying something like this: “You are only getting this result because you are assuming that if platonism is true, then plenitudinous platonism is true (i.e., that if abstract objects exist at all, then there is a plenitude of abstract objects). Given this assumption, it is of course true that the platonistic existence facts do not do anything to determine which mathematical sentences are good and which ones are bad. But if we drop this assumption, then the existence facts will play an important role in determining which mathematical sentences are good and which ones are not.” But I think this is just false; in this scenario, the platonistic facts would play a role in determining which mathematical sentences were *true*, but they still would not play any role in determining which ones were *good.* Suppose that at midnight, God created the natural numbers but not the real numbers. Then, e.g., ‘3 is positive’ would be true and ‘3.5 is positive’ would be false; but there would not be any interesting sense in which the former was good and the latter was bad. ‘3.5 is positive’ would still be FAPP-true, and it would still be *good* in all the ways that really matter. The right thing to say in this scenario would be that since God created only some of the possible abstract objects, only some of the good mathematical sentences became true.

So it seems to me that all of us—even platonists—should endorse something like the FAPP-ist view of the objective correctness of our mathematical theories. And this gives us a powerful argument for the claim that FAPP-ism is true and that error theorists can use it to respond the objective-correctness argument against their view.

4 Objections and Responses

In this section, I will quickly respond to four worries about FAPP-ist error theory.

4.1 Intended Parts of the Mathematical Realm

Consider the following objection to plenitudinous platonism and FAPP-ist error theory:

> There are pairs of mathematical theories—e.g., Zermelo-Frankel set theory plus the continuum hypothesis (ZF+CH) and Zermelo-Frankel set theory plus the negation of the continuum hypothesis (ZF+~CH)—such that (a) both of the theories are internally consistent, but (b) they are inconsistent with each other. But this creates a problem for plenitudinous platonism and

> FAPP-ist error theory. For since ZF+CH and ZF+~CH are both internally consistent, it would seem to follow that both theories *could be true* and that the objects that they describe *could exist.* But if these objects *could* exist, then according to plenitudinous platonism, they *do* exist. But if both of these domains of objects exist—i.e., if the objects described by ZF+CH exist and the objects described by ZF+~CH also exist—then it seems that ZF+CH and ZF+~CH are *both true.* And so plenitudinous platonism seems to lead to contradiction; for it seems to imply that ZF+CH and ZF+~CH are both true. And if this is right, then FAPP-ist error theory is in trouble too—for it leads to the undesirable result that ZF+CH and ZF+~CH are both FAPP-true.

Let me begin by explaining how I think plenitudinous platonists should respond to this objection, and then I will explain how FAPP-ist error theorists can respond in an essentially equivalent way.

The problem with the objection to plenitudinous platonism is that that view does *not* entail that ZF+CH and ZF+~CH are both true. Rather, it entails that they both *accurately describe collections of abstract objects*, or parts of the mathematical realm. But as I have argued elsewhere (1998, 2009), plenitudinous platonists should not say that every purely mathematical theory that accurately describes a collection of abstract objects is true. Rather, they should define mathematical truth in terms of accurately describing the *intended* objects—or the intended parts of the mathematical realm. More precisely, I argued in my (2009) that plenitudinous platonists should endorse the following view of mathematical truth:

> [T] A pure mathematical sentence S is *true* if and only if it is true in all the parts of the mathematical realm that count as intended in the given branch of mathematics (and there is at least one such part of the mathematical realm); and S is *false* if and only if it is false in all such parts of the mathematical realm (or there is no such part of the mathematical realm[14]); and if S is true in some intended parts of the mathematical realm and false in others, then there is no fact of the matter whether S is true or false.[15]

14 We actually do not need this parenthetical remark because if there is no such part of the mathematical realm, then the claim that S is false in all such parts will be vacuously true.

15 If plenitudinous platonists endorse [T], then they will have to say that there could be bivalence failures in mathematics. Suppose, e.g., that our full conception of the universe of sets is not perfectly precise and, in particular, that ZF+CH and ZF+~CH are both perfectly consistent with our full conception of the universe of sets. Given this background assumption, it is easy to argue that if plenitudinous platonism and [T] are both true, then there is no fact of the matter whether CH is true or false. But I do not think this is a problem; indeed, I think this is exactly what we *should* say if our full conception of the universe of sets is imprecise in the above way. For more on this, see my (2009).

So while plenitudinous platonism entails that all internally consistent purely mathematical theories accurately describe parts of the mathematical realm, it does *not* entail that all such theories are true. Consider, e. g., the sentence '16 does not have a successor.' This sentence is internally consistent, and so it follows from plenitudinous platonism that it accurately describes some mathematical structure. But it *does not* accurately describe the *natural-number* structure, and so (if we interpret this sentence according to the rules of ordinary English) it is *false.* And if you uttered this sentence intending to say something about the natural numbers (and if you were speaking English), then your utterance would be false. And if plenitudinous platonists endorse [T], as I think they should, then they can say that utterances like this are false. So, again, plenitudinous platonism does not entail that all purely mathematical theories that are internally consistent are true. In particular, it does not entail that ZF+CH is true, and it does not entail that ZF+~CH is true. And so, it obviously does not entail that they are *both* true. And so the above objection to plenitudinous platonism fails.

Similar remarks can be made about the objection to FAPP-ist error theory. If I am right that plenitudinous platonists should endorse [T], then FAPP-ist error theorists should endorse the following:

> [FT] A pure mathematical sentence S is FAPP-true if and only if the following is true: if plenitudinous platonism had been true (i. e., if there had been a plenitudinous realm of abstract mathematical objects), then (a) S would have been true in all the parts of the mathematical realm that would have counted as intended in the given branch of mathematics, and (b) there would have been at least one such part of the mathematical realm. And S is *FAPP-false* if and only if S would have been false in all intended parts of the mathematical realm, if plenitudinous platonism had been true. And if S would have been true in some intended parts of the mathematical realm and false in others (if plenitudinous platonism had been true), then there is no fact of the matter whether S is FAPP-true or FAPP-false.

Thus, since FAPP-ist error theorists can (and, I think, *should*) endorse [FT], their view does not entail that ZF+CH and ZF+~CH are both FAPP-true. Indeed, it does not entail that *either* of them is FAPP-true.

4.2 Possible Worlds

Consider, next, this objection to FAPP-ist error theory:

> (a) Counterfactuals are claims about possible worlds, and (b) possible worlds are abstract objects,[16] and so (c) anti-platonists cannot make any progress by switching from '3 is prime' to [CF].

I have two things to say in response to this worry. First, according to error theorists of the sort I have in mind, the counterfactual conditional at work in counterfactuals like [CF] is a *primitive*, and so these sentences do not refer to or quantify over possible worlds. Note, however, that I am *not* claiming here that *all* counterfactuals can be understood as employing a primitive counterfactual conditional. I am talking only about the counterfactuals that error theorists of the sort that I have in mind commit to—i. e., counterfactuals like [CF]. Moreover, I am not making any claim here about *ordinary-language* counterfactuals. If ordinary-language counterfactuals are best interpreted as being about possible worlds (and, for whatever it is worth, I doubt this), then my response will be to introduce a *novel* counterfactual operator—'If it had been the case that…then it would* have been the case that…'—and just *stipulate* that (a) this operator is a primitive; and (b) sentences of the form 'If it had been the case that P, then it would* have been the case that Q' do not make any claims about possible worlds (unless P and Q themselves make claims about possible worlds); and (c) this is the counterfactual operator that is at work in the counterfactuals that error theorists are committed to.

Second, I think it can be argued—and, indeed, I *have* argued—that counterfactuals like [CF] are *metaphysically innocent*; more specifically, I argued in my (2021, chapter 6) that *nothing is required of reality* to make these counterfactuals true; in particular, I argued that even if there are no such things as possible worlds, counterfactuals like [CF] are still true. But, unfortunately, I do not have the space to argue for this here.

4.3 Counterpossibles

A third worry you might have about FAPP-ist error theory can be put like this:

16 The claim that possible worlds are abstract objects is, of course, controversial; so you might think that FAPP-ist error theorists could respond to the present objection by rejecting this view of possible worlds; but I will not pursue this avenue of response here.

> Error theorists should say not just that abstract objects *do not* exist but that they *could not* exist—and so FAPP-ist error theorists should say that counterfactuals like [CF] are actually *counterpossibles.* But you might think this is problematic because you might think counterpossibles are all vacuously true (e. g., Williamson (2007) argues for this vacuist view).

I have two responses to this. First, error theorists do not have to say that the existence of abstract objects is impossible, and indeed, I (and others) have argued elsewhere—see, e. g., Field (1989), Hellman (1989), and my (1998, 2021)—that the best versions of anti-platonism are contingentist views that say that the existence of abstract objects is possible. Second, I do not think the vacuist view of counterpossibles is right. That view is badly counterintuitive, and it is incompatible with the fact that we make widespread use of counterpossible reasoning. I think that nowadays most philosophers endorse the non-vacuist view that at least some counterpossibles are non-vacuously true and false. In any event, there are lots of papers that develop ways to understand counterpossibles in non-vacuous ways; see, e. g., Mares and Fuhrmann (1995), Nolan (1997), Brogaard and Salerno (2013), Bjerring (2014), and Bernstein (2016). Also, for an argument against Williamson's vacuist view, see, e. g., Berto, French, Priest, and Ripley (2018).

4.4 Counterfactuals Involving Reference Failures

A fourth objection to FAPP-ist error theory is based on the worry that if platonism is not true, then counterfactuals like [CF] are not true because they contain singular terms (like '3') that do not refer to anything. I have two responses to this. First, I think that the truth of [CF] is actually perfectly compatible with the claim that '3' does not refer. For (a) if the antecedent of [CF] had been true, then '3' would have had a referent, and (b) that referent would have been prime, and so (c) if the antecedent of [CF] had been true, then its consequent would have been true as well. Second, even if the truth of [CF] is *not* compatible with the claim that '3' does not refer, it would not undermine my argument in any important way because error theorists could just switch from relying on counterfactuals like [CF] to relying on counterfactuals like

> [CF*] If there had been a plenitudinous mathematical realm, and if everything else (including our linguistic intentions) remained the same, then '3 is prime' sentence tokens (literally intended and literally interpreted) would have been true.

So errortheorists do not need to rely on counterfactuals like [CF]. But I think the truth of [CF] is compatible with the claim that '3' does not refer, so I think it is fine for error theorists to rely on counterfactuals like [CF].

5 The Quine-Putnam Argument Against Error Theory

Consider the following argument for the truth of our mathematical theories—and, hence, against mathematical error theory:

> *The Quine-Putnam Argument:* We need to acknowledge that our mathematical theories are true because (a) they are embedded (or ineliminably embedded) in our scientific theories, and (b) our scientific theories are true.[17]

I responded to this argument in depth in my (1996) and (1998). I will not attempt a full response to the Quine-Putnam argument here, but I would like to say a few words about it. Very roughly, I think that error theorists can respond to the Quine-Putnam argument by saying something like the following:

> If there are any such things as abstract objects, then they are causally inert. But given this, it follows that the truth of our empirical theories depends on two sets of facts that hold or do not hold independently of one another. One of these sets of facts is purely platonistic and mathematical, and the other is purely physical and anti-platonistic.[18] Since these two sets of facts hold or do not hold independently of one another, we can maintain that (a) there does obtain a set of purely physical facts of the sort required here, i.e., the sort needed to make our empirical theories true, but (b) there does not obtain a set of purely platonistic facts of the sort required for the truth of our empirical theories (because there are no such things as abstract objects). Therefore, mathematical error theory is consistent with an essentially realistic view of empirical science because we can maintain that even if there are no such things as mathematical objects and, hence, our empirical theories are not strictly true, these theories still paint an essentially accurate picture of the physical world because the physical world is just the way it needs to be for our empirical theories to be true. In other words, we can maintain that the physical world holds up *its end* of the empirical-science bargain. Here is another way to put all of this: we can say that while our empirical theories are not strictly true, they are still correct in an important sense of the term because they are *FAPP-true*—i.e., they are such that they would have been true if there had been a plenitudinous realm of abstract objects. The reason that we (error theorists) can endorse this stance is that abstract objects would (if they existed) be causally inert; because of this, it would not make any difference to the physical world if the whole plenitude of abstract objects suddenly popped into existence. This is why we can say that our empirical theories are FAPP-true. And so, in sum, error theorists can endorse the very same view of the correctness of our empirical theories that they endorse of the correctness of our mathematical theories.

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17 For articulations of this argument, see Quine (1948, 1951), Putnam (1971), Resnik (1997), Colyvan (2001), and Baker (2005, 2009).

18 It does not follow from what I am saying here that we can always separate out the content of mixed sentences—i.e., sentences that refer to (or quantify over) abstract and concrete objects.

I think this gives error theorists an adequate response to the Quine-Putnam argument. But one might press error theorists here by asking them why our empirical theories make reference to mathematical objects in the first place, given that those objects do not really exist. Why is this *helpful?* After all, it is not helpful in physics to refer to the Easter Bunny. So why is it helpful to refer to numbers?

But there is an obvious way for error theorists to respond to these questions. They can say that mathematics functions in empirical science as a descriptive aid; more precisely, it gives us an easy way to make certain kinds of claims about the physical world. For instance, by using singular terms that refer (or *purport* to refer) to real numbers, we give ourselves an easy way to describe the temperature states of physical systems. In essence, the numerals serve as *names* of the possible temperature states. Instead of using names like 'Fred' and 'Barney' to refer to these states, we use names like '32 degrees Fahrenheit'; and this is very convenient because the possible temperature states are lined up (i. e., *structured*) in the same way that the real numbers are lined up (or would be lined up, if they existed).

This theory of the role that mathematics plays in empirical science also explains why it would not matter to our empirical scientific endeavors if mathematical objects did not actually exist so that our mathematical theories were not strictly true. The reason this would not undermine our scientific endeavors is that mathematics can do what it is supposed to do in empirical science—i. e., it can succeed in its role as a descriptive aid—even if it is not true. For instance, even if there are no such things as real numbers, it is still helpful to use real-number expressions to talk about temperature states. Indeed, it is easy to see that the question of whether real numbers actually exist is totally irrelevant to the usefulness of real-number expressions to temperature talk. If I tell you that it is 30 degrees Fahrenheit outside, I succeed in communicating something to you about the air outside, even if what I said is not strictly true because the number 30 does not really exist. This, in a nutshell, is because (a) my claim is FAPP-true (assuming that the air outside is, in fact, the right temperature, i. e., the temperature picked out by the expression '30 degrees Fahrenheit'); and (b) since numbers would be causally inert if they existed, it just does not matter to the state of the air outside whether numbers actually exist. It is not as if we think that the number 30 is somehow making the air outside be the temperature that it is. That number could pop in and out of existence, and nothing about the air would change. The sentence 'It is 30 degrees outside' would flip back and forth between being true and being false, but it would remain FAPP-true through all of this, and we would not notice the flip-flopping; indeed, it would not matter to us at all. In short, since we are essentially just using the expression '30 degrees' as the name of a certain temperature state, it follows that the sentence 'It is 30 degrees outside' would have the exact same amount of usefulness to us regardless of whether numbers existed and regardless of whether this sentence was literally true.

(This is all very quick; for a more thorough response to the Quine-Putnam argument, see my (1996) and (1998).)

6 Saving Wittgenstein's Insights … and Avoiding His Mistakes

So far, I have argued that FAPP-ist error theory is a defensible, plausible view. I now want to explain how this view enables us to (a) preserve a number of Wittgenstein's insights about mathematics, while (b) avoiding a mistake that he made about mathematics.

6.1 Saving Wittgensteinian Insights

FAPP-ist error theory enables us to save a number of plausible claims that Wittgenstein made about mathematics. In this subsection, I will discuss a few of them.

6.1.1 Anti-Platonism

FAPP-ist error theory entails that platonism is false—i. e., that there are no such things as mathematical objects like numbers and sets. This is obviously something that Wittgenstein believed (see, e. g., his RFM 1956), and it is obviously an entailment of FAPP-ist error theory, so I will not bother to say anything more about this here.

6.1.2 The Sense in Which Mathematical Theories Are True

Wittgenstein did not think that our mathematical theories are true in any ordinary straightforward sense. He thought that we can *say* that mathematical claims are true and false but that, if we do this, we use the words 'true' and 'false' differently from how we use them in connection with ordinary empirical sentences like 'Mars is round.' (Wittgenstein makes claims like this in multiple places, but see, most notably, §6 of the *Tractatus* (1921).)

FAPP-ist error theory gives us this same result, and it gives us a precise account of what ‘true’ and ‘false’ mean if we speak this way. For if we like, we can use ‘true’ in mathematics to mean *FAPP-true*, and we can use ‘false’ to mean *FAPP-false* (see §4.1 for a definition of ‘FAPP-false’).

6.1.3 Mathematics as a Human Invention

Wittgenstein believed that mathematics is a human invention.[19] And this is a clear entailment of FAPP-ist error theory. For according to FAPP-ist error theory, which mathematical sentences are correct is completely determined by *our intentions*—or by the mathematical structures that *we* have in mind.

The beauty of FAPP-ist error theory—or one of its many beauties—is that it delivers this sense in which mathematics is a human invention while also saving the obvious fact that there is something objective and factual about mathematics. There is clearly some objective sense in which ‘3 is prime’ is *correct* and ‘4 is prime’ is *incorrect.* For FAPP-ist error theorists, this boils down to a mixture of (a) objective logical facts (about what follows from plenitudinous platonism) and (b) objective facts about our intentions—about what we as a society mean by our mathematical words.

6.1.4 Mathematical Relativism

Let *mathematical relativism* be the view that different cultures can endorse different mathematical theories that (a) seem to be incompatible with one another but (b) are both true, or correct. Suppose, for instance, that Martians endorse ZF+CH, and we endorse ZF+~CH (or some set theory in which ~CH is provable); according to mathematical relativism, it could nonetheless be the case that *both of our set theories are true, or correct.* Put roughly, the relativist’s idea is that CH could be “true or correct for Martians” even if it is not “true or correct for us”.

It should be obvious that FAPP-ist error theory implies that mathematical relativism is true. For it entails that mathematical correctness is determined by the intentions of the people doing the talking. So if Martians have in mind set-theoretic hierarchies that are characterized by ZF+CH, and if we have in mind set-theoretic hierarchies that are characterized by ZF+~CH, then if we endorse ZF+~CH and they

19 He says this numerous times in his RFM 1956; see, e.g., Part I, §168.

endorse ZF+CH, then we will both be right. For ZF+CH will be correct when uttered by Martians, and ZF+~CH will be correct when uttered by us.

I do not know whether Wittgenstein ever explicitly endorsed mathematical relativism, but it seems like the kind of view that he would have endorsed. At any rate, I think that Wittgensteinians *should* endorse relativism because, as we will see in §7.3, I think mathematical relativism is *true.*

6.2 Avoiding Wittgenstein's Mistakes

I now want to explain how FAPP-ist error theory enables us to avoid an aspect of Wittgenstein's philosophy of mathematics that, I think, was a mistake. Wittgenstein endorsed a *non-face-value* view of the meanings of mathematical sentences. I think this was a mistake because I think that non-face-value interpretations of mathematical discourse are out of touch with the *ordinary usage* of mathematical sentences. If we come at mathematical discourse as empirical linguists, without worrying about ontology, then it seems to me that the obvious interpretation is the platonistic interpretation. This is the interpretation that takes mathematics at *face value.* E. g., it takes '3 is prime' to say that 3 is prime; and it takes 'There are infinitely many primes' to say that there are infinitely many primes; and so on. This just seems obviously *right*; and I gave a quick argument for this face-value interpretation in §2.2.

But, again, Wittgenstein rejected the obvious, face-value interpretation of mathematical discourse; he endorsed a non-face-value theory of the meanings of mathematical sentences. Wittgenstein's view here is subtle and complex, and there are multiple ways to interpret him. My own view is that Wittgenstein is best interpreted as endorsing some version of the view that mathematical sentences *do not say anything,*[20] but in the present context, it does not matter whether this is the right interpretation of Wittgenstein. Indeed, for our purposes here, it does not matter what Wittgenstein's positive view was. All that matters (for our purposes here) is that Wittgenstein *did not* endorse the face-value interpretation of mathematical discourse—and this seems entirely obvious and uncontroversial. The reason this is important is that, as I just pointed out, we have good reason to think that the face-value interpretation of mathematical discourse is *correct.* Indeed, it seems to me that the *only* reason to endorse a non-face-value interpretation is that it enables us to avoid platonism. But as we have seen in this paper, we

20 For example, in §6.2 of the *Tractatus*, Wittgenstein says that "the propositions of mathematics are […] pseudo-propositions."

do not have to endorse a non-face-value interpretation of mathematical discourse in order to avoid platonism; we can endorse the obvious, face value interpretation —i.e., the platonistic interpretation—and still avoid platonism. We can do this by endorsing FAPP-ist error theory.

(I will not pursue this here, but I think that in endorsing a non-face-value interpretation of mathematical discourse, Wittgenstein violated one of his own views about what we ought not to do in philosophy. It seems plausible that, in doing philosophy, we should not take ordinary-language claims out of their natural home and twist their meanings for philosophical purposes. But I think it is arguable that this is just what Wittgenstein did in connection with mathematical discourse; in particular, I think it is arguable that Wittgenstein let his philosophical view—in particular, his anti-platonism—drive him to endorse an interpretation of mathematical sentences that is out of touch with the ordinary *usage* of those sentences. But I will not attempt to argue for this claim here.)

Finally, I think that Wittgenstein made other mistakes about mathematics as well—in addition to the mistake of endorsing a non-face-value interpretation of mathematical discourse—and, once again, I think we can avoid these other mistakes if we endorse FAPP-ist error theory. For example, Wittgenstein's negative remarks about the mathematical work of Cantor and Gödel commit him to a kind of *revisionism*,[21] and I think this is problematic. I think that if a philosophical line of thought implies that there is something wrong with an important mathematical theorem—a theorem that is widely accepted in mathematics—then it is extremely likely that there is something wrong with the philosophical line of thought and not the mathematical theorem. Moreover, in our particular case, I think it is relatively easy to argue that there is no good philosophical objection to the theorems of Cantor or Gödel. I will not attempt to argue for this here, but I would like to point out that if we endorse FAPP-ist error theory, then we can avoid what seems to me to be the main philosophical worry that you might have about Cantor's and Gödel's theorems—namely, that they lead to unacceptable kinds of platonism—*without* endorsing a revisionist view of those theorems. This is because FAPP-ist error theory enables us to endorse Gödel's and Cantor's theorems while still endorsing anti-platonism. More precisely, FAPP-ist error theorists can say that Cantor's and Gödel's theorems are *FAPP-true*—and that this is the good-making feature in mathematics—and they can do this without committing to the existence of any abstract objects.

21 Wittgenstein criticizes Cantor's set theory in multiple places, e.g., his PR 1975; and he criticizes Gödel's theorem in his RFM 1956.

7 Three Consequences of My Arguments

7.1 The Superiority of FAPP-ist Error Theory over Wittgenstein's View

One obvious consequence of my arguments in this paper is that FAPP-ist error theory is superior to Wittgenstein's philosophy of mathematics. This follows from two points. First, FAPP-ist error theory enables us to avoid the Wittgensteinian mistakes discussed in §6.2. And second, the only complaint that one might reasonably have about FAPP-ist error theory is that it implies that mathematical sentences like '2 + 2 = 4' are *not true*; you might complain about this because you might think that such sentences *are* true. In this paper, I have tried to argue that there are, in fact, no good reasons to think that sentences like '2 + 2 = 4' are literally true. But in the present context—i. e., the context of comparing FAPP-ist error theory to Wittgenstein's view—this does not matter. For Wittgenstein's view also implies that these sentences are not straightforwardly true. Now, Wittgensteinians can claim that there is a *sense* in which sentences like '2 + 2 = 4' are "true", but as we have seen, FAPP-ist error theorists can say this too. So FAPP-ist error theorists and Wittgensteinians are on all fours with respect to the truth of mathematics. So Wittgensteinians cannot claim superiority on the only potential weakness of FAPP-ist error theory. But, as we have seen, FAPP-ist error theorists can claim superiority over Wittgenstein's view because, as we saw in §6.2, FAPP-ist error theorists can avoid two different mistakes that Wittgenstein made about mathematics. And so it follows from all of this that FAPP-ist error theory is superior to Wittgenstein's view.

7.2 Platonism with a Human Face

A second consequence of the arguments of this paper is that the best version of platonism, namely, plenitudinous platonism, has some pretty surprising features —features that, I think, make the view a lot more appealing than it would otherwise be. I will just mention two of these features here. It turns out—quite surprisingly, I think—that platonism delivers two of the four Wittgensteinian insights discussed in §6.1. In particular, plenitudinous platonism entails the following two results: (i) there is an important sense in which mathematics is a *human invention*; and (ii) *mathematical relativism* is true. The reason that plenitudinous platonism entails claim (i) is that it entails that *which mathematical sentences are true is de-*

termined by facts about our intentions. And the reason that plenitudinous platonism entails claim (ii) is that it entails claims like the following:

> If Martians endorse ZF+CH (and if they intend to be describing set-theoretic hierarchies that are in fact characterized by ZF+CH), and if we endorse ZF+~CH (and if we intend to be describing set-theoretic hierarchies that are in fact characterized by ZF+~CH), then we are *both right*—i. e., both of our theories are true.

It might be surprising that platonists should endorse mathematical relativism, but it is a straightforward consequence of the plenitudinous platonist view that I have described in this paper.

Given that plenitudinous platonism entails claims (i) and (ii) above, I think it is fair to say that part of what Wittgenstein complained about, vis-à-vis platonism, was based on a confused view of what platonism actually implies. But I suppose we can forgive Wittgenstein here, since just about everyone else has been confused on this point as well—i. e., since very few people appreciate the way in which the best version of platonism entails claims (i) and (ii).

7.3 Invention and Relativism

Finally, it is worth noting that if I am right that plenitudinous platonism is the best version of platonism and FAPP-ist error theory is the best version of anti-platonism, then it follows that claims (i) and (ii) from §7.2 are just *true*—i. e., it follows that mathematical relativism is true and that there is an important sense in which mathematics is invented. For, again, plenitudinous platonism and FAPP-ist error theory both entail claims (i) and (ii).

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